

## Mathematical Modeling for the Distribution of Relief Items under Uncertainty to Control Vehicle Costs and Downtime

Yadollah Gooran Orimi<sup>1</sup> , Ahmad Mehrabian<sup>2✉</sup> , Arash Naderian<sup>3</sup> , Samad Ayazi<sup>4</sup> 

1. Department of Industrial Management, AK.C., Islamic Azad University, Aliabad Katoul, Iran, Email: [y.gooran@iau.ir](mailto:y.gooran@iau.ir)
2. Corresponding author, Department of Industrial Engineering, AK.C., Islamic Azad University, Aliabad Katoul, Iran, Email: [Mehrabian.project@iau.ac.ir](mailto:Mehrabian.project@iau.ac.ir)
3. Department of Accounting, AK.C., Islamic Azad University, Aliabad Katoul, Iran, Email: [naderian@iau.ac.ir](mailto:naderian@iau.ac.ir)
4. Department of Accounting, AK.C., Islamic Azad University, Aliabad Katoul, Iran, Email: [Samadayazi@iau.ac.ir](mailto:Samadayazi@iau.ac.ir)

| Article Info                                                                                                                                             | ABSTRACT                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Article type:</b><br>Research Article                                                                                                                 | This research focuses on designing a mathematical model for the location and routing of humanitarian aid distribution in humanitarian organizations, developed under conditions of uncertainty and with an approach to control costs and vehicle downtime. Aid organizations face challenges such as demand variability, changing geographical conditions, and resource limitations when responding to crises. The goal of this research is to optimize the distribution process of essential items by providing a multi-objective model that simultaneously determines optimal locations and suitable distribution routes by controlling costs and time. To solve the model, the Epsilon Constraint (EC) method and the NSGA-II multi-objective genetic algorithm were used. The results show that the proposed model can select the shortest routes, minimize logistical costs, and optimally allocate vehicles according to their type to each destination. The study's findings indicate that the presented model significantly improves the efficiency and effectiveness of humanitarian logistics operations. The model's validity was examined through a case study, and the results were compared with existing designs, demonstrating that the proposed model's performance is superior to current approaches. Overall, this research provides decision-makers with a scientific and data-driven tool to develop more effective disaster response strategies by reducing costs and optimizing resources. |
| <b>Article history:</b><br>Received 6 May 2026<br>Received in revised form 12 June 2026<br>Accepted 16 September 2026<br>Published online 1 October 2026 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <b>Keywords:</b><br>localization, routing, humanitarian logistics, cost control, downtime                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

**Cite this article:** Orimi, Y. & et al, (2026). Mathematical Modeling for the Distribution of Relief Items under Uncertainty to Control Vehicle Costs and Downtime. *Journal of Engineering Management and Soft Computing*, 12 (4). 59-82.

**DOI:** <https://doi.org/10.22091/jemsc.2026.15874.1376>



© The Author(s) retains the copyright.

DOI: <https://doi.org/10.22091/jemsc.2026.15874.1376>

**Publisher:** University of Qom

## 1) Introduction

Disasters are often unpredictable and lead to extensive human casualties, financial losses, and social and environmental damage (Wang et al., 2021). An example of such a crisis was the COVID-19 outbreak in 2019, which had severe global consequences (Jahangiri et al., 2021). Statistics also show that natural disasters, such as earthquakes and floods, are among the most common catastrophes, claiming thousands of victims. In these situations, humanitarian logistics plays a key role in the fair movement, storage, and distribution of relief items (Jahangiri et al., 2021). Given resource limitations, ensuring equity in allocation is essential so that all regions benefit from aid commensurate with their demand (Anaya-Arenas et al., 2018; Yofrido & Harjana, 2019).

Natural and human-made disasters are occurring with increasing intensity, imposing heavy impacts on communities. In this context, the humanitarian supply chain, as a specific type of supply chain, plays a vital role in mitigating the effects of these crises, aiming to save lives, alleviate suffering, and restore communities to self-sufficiency (Izadi et al., 2025; Jahangiri et al., 2021). According to the WHO definition, crises cause widespread death, destruction, and environmental disruption, requiring inter-regional assistance (Shao-hong et al., 2022). Each year, over 500 crises claim 75,000 lives and affect more than 200 million others. The increased demand in such situations often exceeds available resources, complicating the process of relief management and planning (Peng et al., 2022). The success of relief operations depends on the efficiency of the supply chain (Abdolhossein Zadeh et al., 2025). However, performance evaluation in this area has received less attention; only 45% of relief organizations address this issue, and only 20% implement it consistently (Giedelmann et al., 2022). This is despite the fact that performance measurement can enhance the effectiveness of relief operations and increase accountability (Mamashli et al., 2021; Wei et al., 2020).

Based on the above-mentioned points, this research investigates the problems of emergency facility location and food item distribution through optimal routing during a crisis. The provision of food items refers to all necessary measures, including purchasing operations and the preparation of materials and resources, which must be provided to those affected before, during, and after a crisis. In this context, the location of the disaster, the number of affected areas, the extent of disaster damage, and the demand for emergency food items are highly uncertain (Gorgi et al., 2025). Furthermore, some food items have conditions that may make them highly perishable. Uncertainty, in general, refers to a state that cannot be directly expressed with a definite amount of information. Therefore, uncertainty describes a situation that cannot be specified, or in which different probabilities exist; therefore, each possible state corresponds to a specific possibility. Given that the provision of relief items is one of the basic needs in the early stages of a crisis, effective and efficient distribution must be considered. A cost control approach is a plan or system used to reduce the costs associated with the distribution of relief items in the humanitarian food supply chain. This approach includes optimizing logistical processes, such as warehouse management, correct and efficient transportation methods, production planning, and identifying effective resources.

Given the importance of the above concept, this study, by considering a location-routing problem and defining appropriate functions, aims to provide fair assistance, first to affected locations, and then, to send and distribute through transportation equipment in a humanitarian context to deal with a crisis in a region. For this purpose, using the defined functions, suitable locations for sending relief items are determined (Momen-Vaghefi et al., 2026; Sohrabi et al., 2026). Then, for appropriate

disaster response, decisions are made on how to equip locations using vehicles and through the most suitable route. Additionally, if relief vehicles are dispatched and encounter breakdowns, the timely dispatch of service personnel is arranged to control the repair and service duration of the vehicle, thereby minimizing downtime.

The remainder of the article is organized as follows. Section 2 provides a literature review of prior research. Section 3 presents the mathematical modeling along with the metaheuristic solution method for the problem. Section 4 presents the results of applying the problem in a case study. Finally, Section 5 provides a general conclusion along with suggestions for future research.

## **2) Literature Review**

Two-echelon distribution systems are recognized as one of the important issues in Location Routing Problems (LRP) and have been studied in numerous research studies. In this system, goods are transported from the origin to intermediate facilities and then delivered to final destinations. Various studies have focused on developing diverse models in this area; for example, Yu et al. (2020) presented a multi-objective 2E-LRP model with the NSGA-II algorithm for waste management. Wei et al. (2020) developed a hybrid model based on ant colony optimization, focusing on disaster relief logistics. Cheng et al. (2021) concentrated on natural disaster debris clearance with a multi-period approach and employed a genetic algorithm. Wang et al. (2021) developed the MP-2ELRP model by combining k-means and PSO. Furthermore, Gandra et al. (2021) addressed two-dimensional loading constraints, and Fallahtafti et al. (2021) presented a multi-objective model for cash transportation. Other studies, such as Cao et al. (2021), Mohamed et al. (2022), Du et al. (2022), Heidari et al. (2022), and Fathi et al. (2026), have also focused on biomass resources, distribution networks under uncertainty, dynamic routing, collaborative delivery, and green issues in 2E-LRP. Alongside LRP, the Vehicle Routing Problem (VRP) has also attracted significant attention. Research by Hasanpour Jesri et al. (2022) and Nozari et al. (2022) has provided various mathematical models and heuristic/metaheuristic algorithms to solve VRP under different conditions such as crises, uncertainty, and resource constraints.

Overall, recent studies indicate that combining exact and metaheuristic methods in LRP and VRP problems can be effective in optimizing costs, time, resource allocation, and uncertainty management in normal and critical conditions (Narimani et al., 2024).

In recent years, the integration of resilience and real-time disruptions in humanitarian logistics has gained significant traction. For instance, Khan and Ali (2023) developed a robust multi-objective location-routing design for emergency medical supplies, focusing heavily on minimizing post-disaster dynamic response times. Silva et al. (2024) addressed the equity factor in relief distribution by introducing a stochastic multi-echelon routing model under severe demand fluctuations. Furthermore, Wang and Zhang (2025) examined vehicle breakdowns in critical supply chains, introducing a machine-learning-assisted metaheuristic to optimize fleet repositioning during active crises. More recently, studies have focused on hyper-connected relief networks, utilizing advanced evolutionary algorithms to balance delivery costs against the deprivation costs of affected populations. Based on the studies mentioned above, Table 1 presents a classification of the relevant literature.

Table 1. Classification of Prior Studies

| Solution Method                | Cost Control Constraint             | Capacity Constraint              | Logic         |          | Time Period |        | Objective Function |                  | Model Name   | Publication Year | Author Name            | Sub-Problems | Main Problem |
|--------------------------------|-------------------------------------|----------------------------------|---------------|----------|-------------|--------|--------------------|------------------|--------------|------------------|------------------------|--------------|--------------|
|                                |                                     |                                  | Indeterminate | Definite | Multiple    | Single | Multi-objective    | Single-objective |              |                  |                        |              |              |
| Metaheuristics                 | -                                   | -                                | -             | *        | -           | *      | *                  | -                | 2E-LRP       | 2020             | Yu et al.              | 2E-LRP       | LRP          |
| Metaheuristics                 | -                                   | -                                | -             | *        | *           | -      | *                  | -                | MP-2ELRP     | 2021             | Cheng et al.           |              |              |
| Heuristics                     | -                                   | -                                | -             | *        | *           | -      | -                  | *                | MP-2ELRP     | 2021             | Wang et al.            |              |              |
| Heuristics                     | -                                   | -                                | -             | *        | -           | *      | -                  | *                | 2E-LRP       | 2021             | Anaya-Arenas et al.    |              |              |
| Exact and Metaheuristics       | -                                   | -                                | -             | *        | -           | *      | *                  | -                | MO-2ELRP     | 2021             | Fallahtafti            |              |              |
| Heuristics                     | -                                   | -                                | -             | *        | -           | *      | -                  | *                | 2E-BLRP      | 2021             | Cao et al.             |              |              |
| Benders Decomposition          | -                                   | *                                | *             | -        | -           | *      | -                  | *                | 2E-SMCLRP    | 2022             | Mohamed et al.         |              |              |
| Heuristics                     | -                                   | -                                | -             | -        | -           | *      | *                  | -                | MD-2E-JDLRP  | 2022             | Du et al.              |              |              |
| Epsilon Constraint and NSGA-II | -                                   | -                                | -             | *        | -           | *      | *                  | -                | G-2ECOLRP    | 2022             | Heidari et al.         | VRP          |              |
| Heuristics                     | -                                   | -                                | -             | *        | -           | *      | -                  | *                | MTVRPTW-SDLT | 2020             | Narimani et al.        |              |              |
| PBC                            | -                                   | *                                | -             | *        | -           | -      | -                  | *                | MTVRPTW      | 2021             | Shao-hong et al.       |              |              |
| CPLEX                          | -                                   | -                                | -             | *        | -           | *      | *                  | -                | MO-MVRP      | 2021             | Rezaei Kallaj et al.   |              |              |
| Heuristics                     | -                                   | -                                | -             | *        | -           | *      | *                  | -                | CMVRPDCDTW   | 2022             | Wang et al.            |              |              |
| Heuristics                     | -                                   | -                                | -             | *        | -           | *      | -                  | *                | MTOVRP       | 2022             | Hasanpour Jesri et al. |              |              |
| Fuzzy Neutrosophic             | -                                   | -                                | -             | *        | -           | *      | *                  | -                | MDVRP        | 2022             | Nozari et al.          | VRP-LRP      |              |
| Deterministic and Heuristics   | Cost control through cost reduction | Fair distribution and corruption | *             | -        | *           | -      | *                  | -                | MP-LVRP      | 2026             | This research          |              |              |

In general, based on the reviewed studies shown in Table 1, most studies focus on the use of LRP and VRP models. This indicates that, relying on strong literature in past studies, these categories of models show desirable capabilities in this field. Therefore, in this research, similar to the reviewed studies, we present a location-routing model under uncertainty for the distribution of relief items. Uncertainty is considered because the non-deterministic aspect has been overlooked in most studies. Existing studies in humanitarian supply chains were interested in examining affected areas during the recovery phase. However, rarely has any study focused on the fair rescue process in post-disaster relief distribution during the response phase.

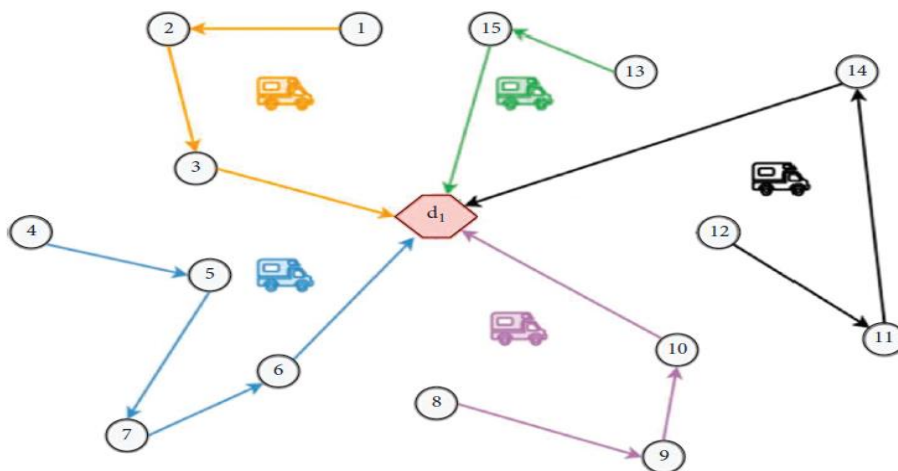
Despite significant advancements in modeling and optimizing LRP and VRP problems, vehicle breakdown time has not been comprehensively investigated as a key factor in the efficiency and effectiveness of these systems. Most existing studies focus on optimizing costs and delivery times, and consequently, the impact of vehicle breakdown time on the overall system performance has been overlooked. In fact, vehicle breakdown time can lead to delays in goods delivery, increased operational costs, and reduced customer satisfaction. Therefore, a significant research gap in this area is to investigate the impact of vehicle breakdown time on the performance of two-echelon distribution systems and vehicle routing.

This research can include the development of new models that consider breakdown time as a key variable, as well as exploring optimization strategies to reduce its negative impacts. Furthermore, using machine learning techniques to predict breakdown time and integrating it with existing models can help improve decision-making in fleet management and route optimization. This research gap can help researchers and industry managers gain a better understanding of the real challenges in

distribution and routing systems and provide more effective solutions to improve the performance of these systems.

### 3) Research Method

This article presents a mathematical model for the precise routing design of emergency vehicles to cope with crises in a humanitarian supply chain. This mathematical model aims to achieve the innovation of minimizing all costs, including the costs of emergency facility failures, fixed costs for establishing shelters, transportation costs for emergency supplies, and service costs, and minimizing the total time for evacuating victims from all affected areas to designated shelters. Additionally, the proposed model provides a multi-objective framework for allocating essential items to all affected destinations by covering various origins in a way that reduces allocation costs and the number of possible allocation routes. The proposed model is presented to achieve the innovation mentioned in this research, which is to allocate essential items fairly to all destinations using the shortest possible route to all destinations, thereby reducing allocation costs. The presented model is a multi-objective linear integer programming model with two objective functions to minimize route costs and the shortest route. The designed supply chain includes several cities, vehicles, and emergency routes, where emergency services are transported by vehicles from specified cities to the disaster site via the best routes at the lowest cost. According to the proposed model, the delivery of emergency services is carried out using only one type of vehicle with the lowest cost and shortest route, such that one city may be covered by several vehicles. Furthermore, the delivery of emergency services is carried out using only one type of vehicle with the lowest cost and shortest repair time. The multi-objective model is converted into a single-objective LP model using the epsilon-constraint method. This section describes all aspects of the deterministic mathematical model developed for the humanitarian supply chain, including the variables, parameters, functions, and constraints. The model can be used for routing emergency vehicles and locating emergency stations, and it also incorporates facility downtime and uncertainty. We aim to perform emergency services in the presented model for five cities with four emergency vehicles, where cities are considered as destinations and vehicles as origins.



**Figure 1.** Overall Mechanism of the Humanitarian Supply Chain

**Symbolization Parameters:***K*: Total number of rescue vehicles*N*: Number of clients*L*: Number of service workers*k*: Number of possible routes*i*: All possible origins*j*: Number of possible destinations*c<sub>ij</sub>*: Current cost on route *i* to *j**t<sub>ij</sub>*: Travel time on route *i* to *j**m<sub>i</sub>*: Demand at node *i**q<sub>k</sub>*: Capacity of rescue vehicle *k**e<sub>i</sub>*: Earliest possible arrival time at node *i**l<sub>i</sub>*: Latest possible arrival time at node *i**f<sub>i</sub>*: Vehicle repair time at node *i**r<sub>k</sub>*: Maximum route time that vehicle *k* is allowed to travel*cr<sub>i</sub>*: Number of vehicle breakdowns at node *i**c'<sub>ij</sub>*: Service cost on route *i* to *j**F<sub>K</sub>*: Fixed cost of establishing route *k* for destination*W<sub>ij</sub>*: Flow between origin *i* and destination *j**C<sub>ik</sub>*: Communication cost between origin *i* and route *k**C<sub>km</sub>*: Communication cost between axes *k* and *m**C<sub>mj</sub>*: Communication cost between axis *m* and destination *j**α*: The coefficient of communication between paths. This coefficient facilitates communication between paths*β*: The coefficient of communication time between paths, representing the maximum allowable time interval for establishing communication between a source and destination pair (*i*, *j*)*γ<sub>i</sub>*: The maximum cost of establishing communication between center *i* and an existing path*d<sub>ik</sub>*: The spatial distance between source *i* and path *k**d<sub>km</sub>*: The spatial distance between paths *k* and *m**d<sub>mj</sub>*: The spatial distance between paths *m* and *j**v<sub>ijkm</sub>*: If the source destination (*i*, *j*) is covered by paths *k* and *m*, then this parameter will be one; otherwise, it will be zero**Definition (1)- *v<sub>ijkm</sub>* in the Model:** If  $C_{ik} + \alpha C_{km} + C_{mj} \leq \gamma_i$ , then  $V_{ijkm} = 1$ , otherwise  $V_{ijkm} = 0$ .**Variables:** The variables in this study are classified into two categories: continuous and integer variables. **Continuous variables are:***T<sub>i</sub>*: Time to enter node *i**w<sub>i</sub>*: Waiting time for vehicle repair at node *i*The variable *w<sub>i</sub>* is important to us because in this study we want to consider the risk of facility failure.**Integer variables are:***x<sub>ijk</sub>*: A binary variable that can be assigned the value zero or oneIf *x<sub>ijk</sub>* is equal to one, there is a route from node *i* to node *j* with vehicle *k* for rescue; otherwise, it

$x_{ijkl}$ : A binary variable that can be assigned the value zero or one

If  $x_{ijkl}$  is equal to one, on the route from node  $i$  to node  $j$ , vehicle  $k$  needs repair; otherwise, it is zero

$x_{ijkm}$ : A binary variable that can be assigned the value zero or one

If the flow from source  $i$  to destination  $j$  is carried out via routes  $k$  and  $m$ , it is equal to one, and zero otherwise.

$Y_k$ : It is equal to one if route  $k$  is chosen, and zero otherwise

$Z_{km}$ : It is equal to one if routes  $k$  and  $m$  are considered simultaneously; otherwise, it is zero.

$Ve_{lkm}$ : It is equal to one if vehicle  $l$  is considered on route  $k$  and  $m$ ; otherwise, it is zero.

## Objective Functions

In this section of mathematical modeling, the components of the bi-objective programming problem, which has two objective functions, are introduced. The objective functions in this research are: minimizing both the transportation cost on each route and the vehicle repair time during humanitarian aid in the humanitarian supply chain. The details of these objective functions are shown in equations (1) and (2).

$$\min \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^K c_{ij} x_{ijk} + c'_{ij} x_{ijkl} \quad (1)$$

The first objective ensures that the travel cost from route  $i$  to  $j$  by vehicle  $k$  is minimized.

$$\min \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^K w_i c r_i x_{ijk} \quad (2)$$

The second objective function ensures that the repair time for each vehicle  $k$  encountering a breakdown on route  $i$  to  $j$  is minimized. By determining the shortest possible repair time for vehicles, we effectively reduce the risk of prolonged breakdowns and service interruptions. In this study, the epsilon-constraint method has been adopted to solve the bi-objective problem. According to this method, one of the objective functions is considered as a constraint by setting an upper bound. In this study, the second objective function  $\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^K w_i c r_i x_{ijk}$  is considered with the upper bound  $f_i$  as a constraint in the form of  $\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^K w_i c r_i x_{ijk} \leq f_i$ . Therefore, when this constraint is added to the set of constraints, it ensures that the total repair time of each vehicle does not exceed the repair time of the vehicle at node  $i$ .

$$\min \sum_{k=1}^K F_k Y_k \quad (3)$$

The first function ensures that the cost of path allocation  $k$  is minimized.

$$\min \sum_{k=1}^K Y_k \quad (4)$$

The second function guarantees that the shortest path  $k$  for relief is obtained.

## Constraints

$$\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^K x_{ijk} \leq K \quad (5)$$

$$\sum_{j=1}^N x_{ijk} = 1 \quad i = 0; k = 1, \dots, K \quad (6)$$

$$\sum_{j=1}^N x_{jik} = 1 \quad i = 0; k = 1, \dots, K \quad (7)$$

$$\sum_{k=1}^K \sum_{j=0}^N x_{ijk} = 1 \quad j \neq i \quad (8)$$

$$\sum_{k=1}^K \sum_{i=0}^N x_{ijk} = 1 \quad i \neq j \quad (9)$$

$$\sum_{k=1}^K \sum_{j=0}^N x_{ijk} (t_{ij} + f_i + w_i) \leq r_k \quad j \neq i \quad (10)$$

$$T_0 = W_0 = f_0 = 0 \quad (11)$$

$$e_i \leq (T_i + w_i) \leq l_i \quad (12)$$

$$x_{ijkm} \leq Y_k \quad \forall i, j, k, m \quad (13)$$

$$x_{ijkm} \leq Y_m \quad \forall i, j, k, m \quad (14)$$

$$\sum_{k=1}^K \sum_{m=0}^M V_{ijkm} x_{ijk} \geq 1 \quad \forall i, j \quad (15)$$

$$\sum_{k=1}^K \sum_{m=0}^M V_{elkm} x_{ijk} \geq 1 \quad \forall k \quad (16)$$

$$VR_{ij}^1 = \{k | d_{ik} + d_{kj} \leq \beta\} \quad (17)$$

$$VR_{ij}^2 = \{(k, m) | (k < m), (k, m \notin VR_{ij}^1) d_{ik} + \alpha d_{km} \leq \beta\} \quad (18)$$

or

$$VR_{ij}^2 = \{(k, m) | (k < m), (k, m \notin VR_{ij}^1) d_{im} + \alpha d_{mk} \leq \beta\}$$

$$Z_{km} \leq Y_k \quad \forall k, m \quad k < m \quad (19)$$

$$Z_{km} \leq Y_m \quad \forall k, m \quad k < m \quad (20)$$

$$x_{ijkm} \in \{0, 1\} \quad \forall i, j, k, m \quad (21)$$

$$Y_k \in \{0, 1\} \quad \forall k \quad (22)$$

$$Z_{km} \in \{0, 1\} \quad \forall k, m \quad k < m \quad (23)$$

$$V_{elkm} \in \{0, 1\} \quad \forall k, m \quad k < m \quad (24)$$

Equation (5) ensures that the selected routes must never exceed the number of vehicles. To maximize coverage, the presence of this constraint in the set of equations is essential. Otherwise, part of the route will not be covered. Equation (6) ensures that the start of the relief route for vehicle  $k$  begins from node zero (origin). Additionally, equation (7) ensures that there is a round trip path between two nodes. Equations (8) and (9) ensure that the selected paths from  $i$  to  $j$  start from origin zero. It also ensures that no path starts from any node and ends at the same node. Equation (10) ensures that the total travel time between two nodes, the vehicle repair time at each node, and the waiting time for service to perform repairs, does not exceed the maximum time that the vehicle is allowed to travel on path  $i$  to  $j$ . Constraint (11) ensures that the total travel time between two nodes starting from the origin node, the vehicle repair time at the origin node, and the waiting time for service to perform repairs at the origin node is equal to zero. Considering  $t_{ij} + f_i = T_i$ , constraint

(12) ensures that the duration of travel, repair, and waiting time for receiving vehicle repair services does not exceed the earliest and latest times. Constraints (13) and (14) indicate that if the connection between cities (i, j) is established through paths (m, k), then these cities must be the origin. Constraint (15) represents the coverage condition for allocation. That is, according to definition (1), if  $C_{ik} + \alpha C_{km} + C_{mj} \leq \gamma_i$ , then  $V_{ijkm} = 1$ , otherwise  $V_{ijkm} = 0$ . Constraint (16) represents the condition for covering each vehicle type for vehicle assignment to each route. Constraint (17) represents the first set of valid routes for each vehicle in the multi-route assignment model. It also represents the second set of valid routes in the multi-route assignment model. Constraint (18) is an either-or constraint. Between the two constraints for the second set of valid routes, only one is always active. For this purpose, we formulate it as follows by adding  $M > 0$  as a large positive number and  $y_1, y_2$  as binary variables, and we add constraints (25) to (28) to the above set of equations.

$$VR_{ij}^2 = \{(k, m) | (k < m), (k, m \notin VR_{ij}^1) d_{ik} + \alpha d_{km} \leq \beta + My_1 \quad (25)$$

$$VR_{ij}^2 = \{(k, m) | (k < m), (k, m \notin VR_{ij}^1) d_{ik} + \alpha d_{km} \leq \beta + My_2 \quad (26)$$

$$y_1 + y_2 = 1 \quad (27)$$

$$y_1, y_2 \in \{0, 1\} \quad (28)$$

Constraints (19) and (20) indicate that paths m and k have been selected; therefore, a communication is established between them and the cities. Constraints (21) to (24) ensure that the values of the decision variables are zero or one.

The above equations are intended for use in situations where people in five crisis-stricken cities need to receive aid. As shown in Figure 2, the flow of aid is established through two-way paths between every two cities. Considering the set of above equations, we want to determine the aid routes and the coverage of each city by vehicles on these routes.

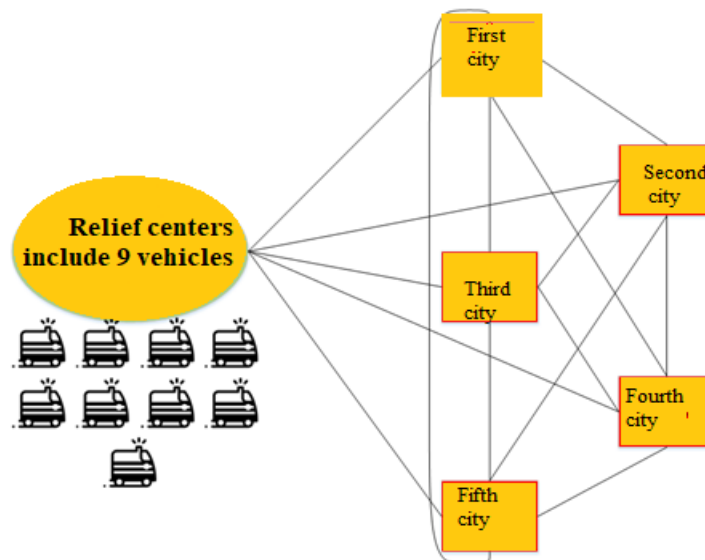


Figure 2. All Relief Routes to Damaged Cities by Vehicles

### 3-2) Incorporating Uncertainty in the Model

In this model, since the required demand for rescue is not predetermined (number of beneficiaries), the demand quantity is considered fuzzy to make the model closer to the real world. Therefore, to achieve innovation related to data uncertainty, we intend to consider the demand parameter for a crisis-stricken area as uncertain due to its proximity to reality. This assumption was overlooked in the basic model of this research presented by Rezaei Kallaj et al. (2021). In fact, we aim to provide a plan to cover unknown demand within the formed supply chain. In this research, we assume that over a fixed and specified period, the total demand  $m_i$  is assumed to be  $S_t$ . We assume that the demand in period  $t$  is not continuously certain and is subject to uncertainty based on real conditions. Uncertainty in the relief set can arise from various factors, including a lack of knowledge regarding the number of injured, limited access to the affected area due to damage, or restricted vehicle capacity. This parameter is considered as an uncertain distance  $[\tilde{S}_t - \hat{S}_t, \tilde{S}_t + \hat{S}_t]$ . According to an uncertain space like  $S_k$ , the symmetric form  $\tilde{S}_t$  is limited as  $\hat{S}_t = \rho \tilde{S}_t$ , where  $\tilde{S}_t$  is the estimated value of the demand parameter. In fact,  $\tilde{S}_t$  is the rate of fluctuation of the demand parameter and  $\rho > 0$  is the level of uncertainty. Therefore, the goal is to estimate a set of random demands in such a way as to provide the most relief at the lowest possible cost. The main decisions in this case of the problem are to estimate the demands and determine their start time. Therefore, we consider a binary variable  $Z_i$  and a variable  $y_{it}$  for each demand. For  $1 \leq c \leq C$ , the value of  $Z_i$  will be one if the demand is satisfied; otherwise, it will be zero. For  $1 \leq t \leq T$  and  $1 \leq i \leq I$ , the value of  $y_{it}$  will be one if the demand for city  $i$  starts in period  $t$ ; otherwise, it will be zero. Since each demand can only occur once in the planning horizon  $H$ , we have:

$$Z_i = \sum_{t=1}^T y_{it}, \quad 1 \leq i \leq I; 1 \leq t \leq T; i = 1, \dots, I; t = 1, \dots, T \quad (29)$$

Equation (29) allows us to transform the variable  $Z_i$  like a continuous variable. Because in equation (29) the independent variables are either zero or one, if we calculate their sum together, we get a value greater than or equal to zero. Now, if a demand is satisfied in period  $k$ , we can calculate  $y_{ik}$  through the continuous variable (30).

$$y_{it} = \sum_{t'=\max\{1,t+1\}}^T y_{it'}, \quad 1 \leq i \leq I; 1 \leq t \leq T; i = 1, \dots, I; t = 1, \dots, T \quad (30)$$

To estimate the demand, we need public donations to meet the demand exceeding the capacity of vehicles  $q_k$ . For this purpose, we consider the variable  $N_{it}$  as the amount of public donations through donors visiting relief centers during period  $t$  for demand  $i$ . If the demand is not met during the period, its value is considered equal to zero. The total donations to estimate the demand should not exceed the available capacity  $q_k$ . Because in this case, food items may be spoiled.

$$y_{it} N_{it} \leq q_k, \quad 1 \leq i \leq I; 1 \leq t \leq T; i = 1, \dots, I; t = 1, \dots, T \quad (31)$$

Therefore, to satisfy the uncertain demand, the above equations are added to the model. The above equations are intended to be used in situations where people in five crisis-hit cities need to receive relief. As shown in Figure 3, the flow of relief is established through two-way routes between the two cities. Considering the above set of equations, we want to determine the relief routes, the time of entry into each of the cities, and the waiting time.

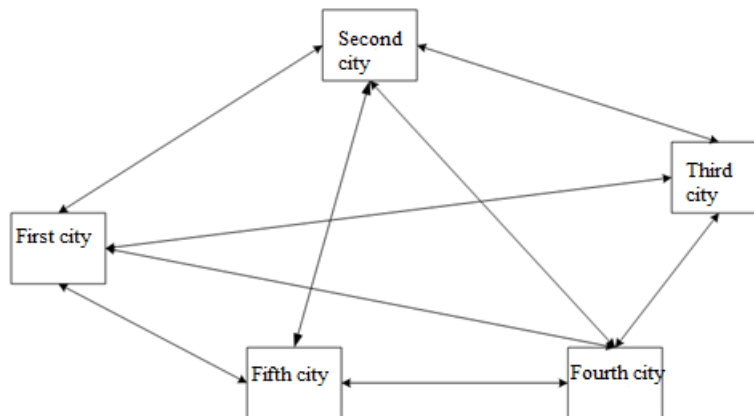


Figure 3. All Relief Routes to the Affected Cities

### 3-3) Problem-Solving Method

#### A) NSGA-II Metaheuristic Approach

In this paper, the NSGA-II metaheuristic algorithm has been used to solve the model. One of the positive points of this algorithm is that, unlike some other algorithms that search the solution space of the problem in only one direction, it searches for the solution in several directions simultaneously. The fact that the genetic algorithm does not require continuity or convexity of the objective function can also be considered another positive feature of this algorithm. The performance of the NSGA-II algorithm is in accordance with the following pseudocode.

**//Start:**

**Step 1:** Generate an initial population

**Begin loop** /\* Loop until a termination condition is met\*/

**Step 2:** Evaluate the fitness of a subset of individuals from the population

**Step 3:** Select pairs of the fittest individuals for reproduction

**Step 4:** Create offspring using a crossover operator

**Step 5:** Introduce genetic variation in the offspring using a mutation operator

**Step 6:** Replace some of the least fit individuals in the current population with the new offspring

**End loop**

**Finish//.**

Additionally, the implementation flowchart is illustrated in Figure 4.

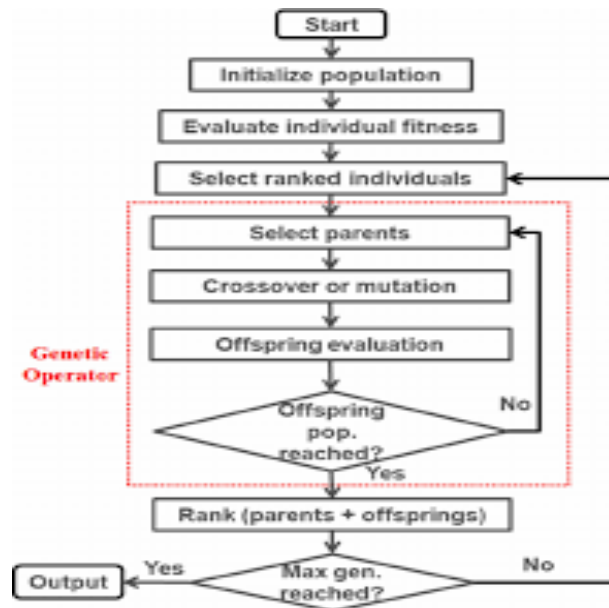


Figure 4. NSGA-II Algorithm Execution Flowchart

### B) Epsilon Constraint Method

In this study, the epsilon constraint method is used to solve the fair item distribution problem. This method transforms the problem into a single-objective linear programming model by selecting one of the objective functions and converting the other functions into constraints. The main advantage of this method over other methods is its applicability in non-convex solution spaces. One of the challenges of exact search-based algorithms is the high computational time, which can be reduced using metaheuristic algorithms. A modified version of this method, recently presented by Abolghasemian et al. (2020), reduces both the search space and the execution time. In this method, the single-objective optimization problem is primarily solved for each objective, and then, the Pareto frontier is estimated.

$$\begin{aligned}
 &\min f_1(x) \\
 &s. t \\
 &f_2(x) \leq \varepsilon_2 \\
 &x \in s
 \end{aligned} \tag{32}$$

## 4) Findings

### 4-1) Model Validity Review

In this section, the validity of the model is reviewed. To this end, the proposed model is implemented in GAMS by specifying dimensions for the sample sizes of the model sets. After implementing the model, the feasibility and boundedness of the mathematical model were evaluated to justify its validity, in accordance with the theorems proven by Abolghasemian et al. (2024). This is because it has been established that the necessary and sufficient condition for the validity of a mathematical model is that it is both feasible and bounded. To implement the model, the size of the sets was first determined in Table 2.

**Table 2.** Sets and Their Size Groups

| Sets                               | Sizes |
|------------------------------------|-------|
| K: Total number of rescue vehicles | 2     |
| N: Number of clients               | 2     |
| L: Number of service workers       | 2     |
| k: Number of possible routes       | 2     |
| i: All possible origins            | 1     |
| j: Number of possible destinations | 3     |

In Table 3, the computational results obtained from the model execution are shown.

**Table 3.** Feasibility and Boundedness of the Mathematical Model

| Ability  | Boundedness | Model solution time (seconds) | Computational tool | Result             |
|----------|-------------|-------------------------------|--------------------|--------------------|
| Scalable | Finite      | 0.059                         | CPLEX              | Mathematical Model |

According to the results presented in Table 3, and the verification of the boundedness and feasibility of the model, the validity of the model is concluded.

#### 4-2) Experimental Results of Solving the Mathematical Model

In this section, the computational experimental results of the proposed model are shown. GAMS software was used to implement the model. The proposed model is able to determine the relief routes. Additionally, it determines the arrival time of each vehicle and the waiting time to receive the necessary repair service. Table 4 shows the service route from each city to another neighboring city based on the solution obtained through zero-one programming. In this table, the transportation cost is considered as the objective function for performing the calculations of the first objective function, while the second objective function is added as a constraint. It is clear that if a route is selected for carrying out relief operations, a value of 1 is assigned to it; otherwise, a value of 0 is assigned.

**Table 4.** Route Allocation for Relief Based on the First Objective Function

| Objective function (currency) | $x_{ij}$    | destination | status |
|-------------------------------|-------------|-------------|--------|
| $f_1^* = 1505$                | First city  | First city  | 0      |
|                               |             | Second city | 0      |
|                               |             | Third city  | 0      |
|                               |             | Fourth city | 1      |
|                               |             | Fifth city  | 1      |
|                               | Second city | First city  | 0      |
|                               |             | Second city | 0      |
|                               |             | Third city  | 1      |
|                               |             | Fourth city | 0      |
|                               |             | Fifth city  | 0      |
|                               | Third city  | First city  | 1      |
|                               |             | Second city | 0      |
|                               |             | Third city  | 0      |

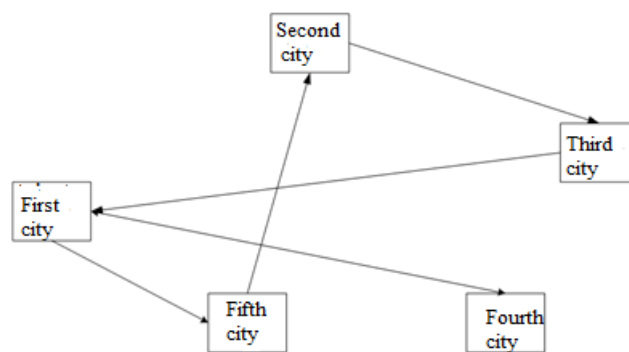
**Table 4.** Route Allocation for Relief Based on the First Objective Function

| Objective function<br>(currency) | $x_{ij}$    | destination | status |
|----------------------------------|-------------|-------------|--------|
|                                  |             | Fourth city | 0      |
|                                  |             | Fifth city  | 0      |
|                                  | Fourth city | First city  | 1      |
|                                  |             | Second city | 0      |
|                                  |             | Third city  | 0      |
|                                  |             | Fourth city | 0      |
|                                  |             | Fifth city  | 0      |
|                                  | Fifth city  | First city  | 0      |
|                                  |             | Second city | 1      |
|                                  |             | Third city  | 0      |
|                                  |             | Fourth city | 0      |
|                                  |             | Fifth city  | 0      |

As can be seen from the results in Table 4, it is not possible to provide relief from a city to itself. In addition, relief to cities 4 and 5 is possible only through city 1. City 2 sends relief service only to city 3 and covers it completely. Part of the relief to city 1 is provided through city 3, and the other part through city 4. Finally, city 5 provides service only to city 2. Table 5 shows the cities providing and receiving service. In addition, Figure 5 shows the optimal route for providing service by relief vehicles under the condition that the first objective function is considered as the objective of the problem in the epsilon constraint method.

**Table 5.** Details of Receiving and Providing Service to the Injured

| City providing relief service | City receiving relief service |
|-------------------------------|-------------------------------|
| First city                    | City 4 and 5                  |
| Second city                   | City 3                        |
| Third city                    | City 1                        |
| Fourth city                   | City 1                        |
| Fifth city                    | City 2                        |

**Figure 5.** Optimal Relief Route to Affected Cities Based on the First Objective Function

Moreover, in Table 6, for calculating the second objective function, i.e., the amount of time it takes to repair vehicles, the first objective function is considered as the objective function of the problem, and the first objective function is added as a constraint in the constraints. It is clear that if a vehicle is on a route that requires service to continue relief operations, it is assigned a value of one, and otherwise, a value of zero.

**Table 6.** Allocation of Service Workers to Continue Relief Operations to the Injured in the Event of a Breakdown

| Objective function<br>(minutes) | $x_{ij}$    | Destination | Status |
|---------------------------------|-------------|-------------|--------|
| $f_2^* = 50$                    | First city  | First city  | 0      |
|                                 |             | Second city | 0      |
|                                 |             | Third city  | 1      |
|                                 |             | Fourth city | 1      |
|                                 |             | Fifth city  | 1      |
|                                 | Second city | First city  | 0      |
|                                 |             | Second city | 0      |
|                                 |             | Third city  | 1      |
|                                 |             | Fourth city | 1      |
|                                 |             | Fifth city  | 0      |
|                                 | Third city  | First city  | 1      |
|                                 |             | Second city | 0      |
|                                 |             | Third city  | 0      |
|                                 |             | Fourth city | 1      |
|                                 |             | Fifth city  | 1      |
|                                 | Fourth city | First city  | 1      |
|                                 |             | Second city | 1      |
|                                 |             | Third city  | 0      |
|                                 |             | Fourth city | 0      |
|                                 |             | Fifth city  | 0      |
|                                 | Fifth city  | First city  | 0      |
|                                 |             | Second city | 1      |
|                                 |             | Third city  | 1      |
|                                 |             | Fourth city | 0      |
|                                 |             | Fifth city  | 0      |

As can be seen in the results of Table 6, the dispatch of service workers to continue the relief operation is as follows:

- If vehicles in city 1 need to be repaired, service workers from the third, fourth, and fifth cities will be sent.
- If vehicles in city 2 need to be serviced, service workers from the third and fourth cities will be sent.
- If vehicles in city 3 need to be serviced, service workers from the first, fourth, and fifth cities will be sent.
- If vehicles in city 4 need to be serviced, service workers from the first and second cities will be sent.
- If vehicles in city 5 need to be serviced, service workers from the second and third cities will be sent.

In Table 7, the details of receiving and providing services based on the second objective function are presented.

**Table 7.** Details of Receiving and Providing Services

| City providing relief service | City receiving relief service |
|-------------------------------|-------------------------------|
| First city                    | City 3 and 4                  |
| Second city                   | City 4 and 5                  |
| Third city                    | City 1, 2 and 5               |
| Fourth city                   | City 1, 2 and 3               |
| Fifth city                    | City 1 and 3                  |

Additionally, the optimal time to enter each city to provide rescue services and the waiting time for vehicle repairs is shown in Table 8. Based on the calculated time lag, it is determined that the maximum possible time to provide rescue services and vehicle repairs is 19 minutes. In Table 8, the time for vehicle arrival and the duration of repair for each of them, if needed, and the time lag are shown.

**Table 8.** Optimal Time for Vehicle Arrival and Repairs

| City        | Arrival time | Repair time | Time forward |
|-------------|--------------|-------------|--------------|
| First city  | 0            | 0           | 0            |
| Third city  | 3            | 16          | 19           |
| Second city | 6            | 13          | 19           |
| Fourth city | 7            | 7           | 14           |
| Fifth city  | 7            | 7           | 14           |

Based on the calculated time advance, vehicles traveling to cities 3 and 2 depart from the target city at time 19, in addition to providing relief service after receiving repair service. This situation occurs for vehicles traveling to cities 4 and 5 at 14 minutes after the start of the process. In addition, by sorting the arrival time in each of the crisis-hit cities in an ascending order, we can determine the order of relief priority. According to the specified priority, the process starts first in the first city, then the third and second cities, and finally the relief process ends by visiting the fourth and fifth cities.

In addition to the above results, the proposed model has also been used for allocation. GAMS software was used to implement the model. The proposed model is able to determine the relief routes, how each city is covered by one or more vehicles, and also the allocation cost. Therefore, firstly, all cities are covered only by relief vehicles that are transported from a specific origin to a specific destination. Therefore, if the vehicle is traveling from origin  $i$  to destination  $j$ , the direct route  $k$  is blocked for any reason and the transfer is impossible, it uses route  $m$  to reach destination  $j$ . Note that if this vehicle uses route  $m$ , it only visits another city but does not stop. This is because that city is supposed to be covered by another relief vehicle. In this case, relief from specific cities of the affected city is not also possible because all cities are involved in a crisis and there is no way to send relief items from one city to another. Therefore, each of the cities is covered by one or more relief vehicles.

Tables 9 to 13 show the shortest possible route for vehicle transport. The service route from the origin to the other affected city is shown using the proposed zero-one programming problem-solving method according to the dispersion search algorithm. Clearly, if the route is selected for vehicle rescue operations, it is assigned a value of one; otherwise, it is assigned a value of zero. According to the specified routes, one or more vehicles are dispatched for rescue.

**Table 9.** Route Allocation for Rescue for the First City

|           | First city | Second city | Third city | Fourth city | Fifth city |
|-----------|------------|-------------|------------|-------------|------------|
| D (start) | 0          | 0           | 1          | 0           | 0          |
| City 3    | 1          |             |            |             |            |
| Route     | D – 3 – 1  |             |            |             |            |

**Table 10.** Route Allocation for Relief for the Second City

|           | First city    | Second city | Third city | Fourth city | Fifth city |
|-----------|---------------|-------------|------------|-------------|------------|
| D (start) | 0             | 0           | 0          | 1           | 0          |
| City 3    | 0             | 1           | 0          | 0           | 0          |
| City 4    | 0             | 0           | 1          | 0           | 0          |
| Route     | D – 4 – 3 – 2 |             |            |             |            |

**Table 11.** Route Allocation for Relief for the Third City

|                  | First city | Second city | Third city | Fourth city | Fifth city |
|------------------|------------|-------------|------------|-------------|------------|
| <b>D (start)</b> | 0          | 0           | 1          | 0           | 0          |
| <b>Route</b>     | D – 3      |             |            |             |            |

**Table 12.** Route Allocation for Relief for the Fourth City

|                  | First city | Second city | Third city | Fourth city | Fifth city |
|------------------|------------|-------------|------------|-------------|------------|
| <b>D (start)</b> | 0          | 0           | 0          | 1           | 0          |
| <b>Route</b>     | D – 4      |             |            |             |            |

**Table 13.** Route Allocation for Relief for the Fifth City

|                   | First city | Second city | Third city | Fourth city | Fifth city |
|-------------------|------------|-------------|------------|-------------|------------|
| <b>D (start)</b>  | 0          | 0           | 1          | 0           | 0          |
| <b>Third city</b> | 0          | 0           | 0          | 0           | 1          |
| <b>Route</b>      | D – 3 – 5  |             |            |             |            |

According to Tables 9 to 13, the shortest route allocation status for relief to each city is as follows:

- Allocation of a vehicle to the first city is possible by passing through the third city.
- Allocation of a vehicle to the second city is possible by passing through the fourth and third cities.
- Allocation of a vehicle to the third city is possible directly from the source of relief to this city.
- Allocation of a vehicle to the fourth city is possible directly from the source of relief to this city.
- Allocation of a vehicle to the fifth city is possible by passing through the third city.

Table 14 presents the lowest possible cost for allocating a route for relief. According to Table 7, the total cost of allocating the shortest route, which is obtained by summing the costs of all possible routes, is equal to 1,492 monetary units.

**Table 14.** Cost of Relief Allocation

| <b>Route<br/>City</b> | <b>D – 3 – 1</b> | <b>D – 4 – 3 – 2</b> | <b>D – 3</b> | <b>D – 4</b> | <b>D – 3 – 5</b> |
|-----------------------|------------------|----------------------|--------------|--------------|------------------|
| <b>First city</b>     | 355              | -                    | -            | -            | -                |
| <b>Second city</b>    | -                | 478                  | -            | -            | -                |
| <b>Third city</b>     | -                | -                    | 120          | -            | -                |
| <b>Fourth city</b>    | -                | -                    | -            | 266          | -                |
| <b>Fifth city</b>     | -                | -                    | -            | -            | 273              |

The relief is carried out by 9 vehicles according to the routes that have been specified. According to Table 15, the types of vehicles and the number at their disposal are specified.

**Table 15.** Types and Number of Relief Vehicles

| <b>ID</b> | <b>Vehicle type</b>                                   | <b>Number</b> |
|-----------|-------------------------------------------------------|---------------|
| Ve1       | Relief items (tents, blankets, etc.)                  | 2             |
| Ve2       | Essential items (food, clothing, mineral water, etc.) | 2             |
| Ve3       | Medical items                                         | 1             |
| Ve4       | Ambulance                                             | 4             |

Table 16 shows the allocation of each vehicle type to each city. This allocation is performed in such a way that each vehicle covers all cities at least once.

**Table 16.** Active Rescue Vehicles on Each Route to Each City

| City        | Vehicle |     |     |     |     |     |     |     | Route         |
|-------------|---------|-----|-----|-----|-----|-----|-----|-----|---------------|
|             | Ve4     | Ve4 | Ve4 | Ve4 | Ve3 | Ve2 | Ve2 | Ve1 |               |
| First city  | 0       | 0   | 1   | 0   | 1   | 0   | 1   | 0   | D – 3 – 1     |
| Second city | 0       | 0   | 0   | 1   | 1   | 1   | 0   | 1   | D – 4 – 3 – 2 |
| Third city  | 0       | 1   | 0   | 0   | 1   | 0   | 1   | 0   | D – 3         |
| Fourth city | 1       | 0   | 0   | 0   | 1   | 1   | 0   | 1   | D – 4         |
| Fifth city  | 0       | 0   | 0   | 1   | 1   | 0   | 1   | 0   | D – 3 – 5     |

Table 16 presents the allocation of each vehicle according to its type. As is evident, in each city, each vehicle type is assigned for coverage based on the specified route.

#### 4-3) Setting the Parameters of the NSGA-II Algorithm

To design experiments in the NSGA-II algorithm, 3 different levels are first defined for its parameters (low with code 1, medium with code 2, and high with code 3). And then, the predefined experiments in this algorithm are run for all possible combinations. The suggested values for the parameters of this algorithm are as shown in Table 17.

**Table 17.** Parameters and Their Levels for the NSGA-II Algorithm

| Parameter                     | Values for each level |         |         |
|-------------------------------|-----------------------|---------|---------|
|                               | Level 1               | Level 2 | Level 3 |
| Population size (PS)          | 50                    | 100     | 200     |
| Crossover rate (CR)           | 0.5                   | 0.7     | 0.9     |
| Mutation rate (MR)            | 0.2                   | 0.3     | 0.5     |
| Maximum iterations (Max_iter) | 100                   | 150     | 200     |

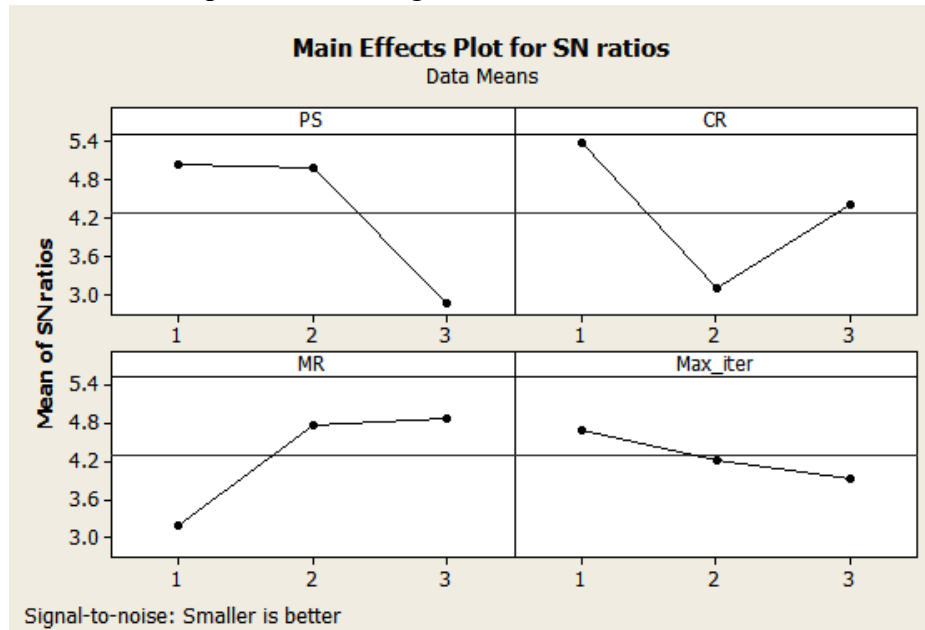
Then, different experiments were created with the L9 Taguchi design, and the NSGA-II algorithm was run for each one. The results of the run are presented in Table 18. This table shows all possible states for different levels considered for the factors of the NSGA-II algorithm. For example, in the first experiment, all factors participate in the experiment for their lowest level. In the second experiment, the PS factor is present with the lowest level value and the other factors are present with their corresponding average level value. Similarly, other possible states are completed based on the permutation rule in statistics. By running each experiment and calculating the value of the MID index, the desired response level is estimated using this index.

**Table 18.** Response Variable Values in the Taguchi Technique for NSGA-II

| Execution number | Algorithm parameters |    |    |          | MID Index |
|------------------|----------------------|----|----|----------|-----------|
|                  | PS                   | CR | MR | Max_iter |           |
| 1                | 1                    | 1  | 1  | 1        | 0.534     |
| 2                | 1                    | 2  | 2  | 2        | 0.612     |
| 3                | 1                    | 3  | 3  | 3        | 0.537     |
| 4                | 2                    | 1  | 2  | 3        | 0.491     |
| 5                | 2                    | 2  | 3  | 1        | 0.567     |
| 6                | 2                    | 3  | 1  | 2        | 0.637     |
| 7                | 3                    | 1  | 3  | 2        | 0.599     |
| 8                | 3                    | 2  | 1  | 3        | 0.973     |
| 9                | 3                    | 3  | 2  | 1        | 0.642     |

Now, by presenting these outputs to the MINITAB software, the signal-to-noise (S/N) ratio chart is presented in Figure 6. Based on the calculated S/N ratio values for all levels considered for each factor, the level that yields the lowest value is selected for that factor. As shown in Figure 6, the lowest value of the S/N ratio for the PS factor occurs when this index is at its high level with code 3.

Therefore, the value that we consider for this parameter in the NSGA-II algorithm will be equal to 200. Additionally, the lowest value of the S/N ratio for the CR index is related to the medium level with code 2 of this factor. Therefore, the CR factor will be present in the algorithm with a value of 0.7. In addition, the lowest value for the MR factor is related to the time when this factor is at its low level with code 1. Therefore, this factor will be present in the algorithm with a value of 0.2. Finally, the Max\_iter factor has the least value relative to the noise when it is at its high level with code 3. Therefore, this factor will be present in the algorithm with a value of 200.



**Figure 6.** Minitab Output for Taguchi Method in NSGA-II Algorithm

Now, based on the output presented in the above graph, the best value of each parameter is specified in Table 19, and other examples are run with these values of algorithm parameters. The optimal value of parameters is presented in Table 19.

**Table 19.** Optimal Value of Variables in NSGA-II

| Parameter                     | Optimum value |
|-------------------------------|---------------|
| Population size (PS)          | 200           |
| Crossover rate (CR)           | 0.7           |
| Mutation rate (MR)            | 0.2           |
| Maximum iterations (Max_iter) | 200           |

#### 4-4) Comparison of Deterministic and Metaheuristic Results

In this part of the research, the results of the proposed model are compared with the metaheuristic algorithm NSGA-II. The efficiency of the algorithm is determined based on two concepts: quality and the response speed. Furthermore, the difference in the value of the objective function of the NSGA-II algorithm is compared with the value of the objective function presented in GAMS, and the speed of solving these two methods is compared. To examine the NSGA-II algorithm used for this problem, this algorithm has been coded in the MATLAB environment. Subsequently, 7 problems in different dimensions have been generated for changes in parameters.

Considering that the gap between the mathematical model and the metaheuristic model has been negligible for small and medium problems, we can trust the results calculated by the metaheuristic model and rely on them to solve large-scale problems.

**Table 20. Results of Solving Sample Problems with GAMS and the NSGA-II Algorithm**

| Problem dimension | Issue number | $L$               | $N$ | $K$ | $i$ | Exact solution with GAMS software |                         | Multi-hybrid genetic algorithm |                         | GAP (%) |
|-------------------|--------------|-------------------|-----|-----|-----|-----------------------------------|-------------------------|--------------------------------|-------------------------|---------|
|                   |              |                   |     |     |     | Objective function                | Solution time (seconds) | Objective function             | Solution time (seconds) |         |
| Small dimension   | PR1          | Problem dimension | 1   | 1   | 1   | 6880                              | 125                     | 6880                           | 57.7                    | 0%      |
|                   | PR2          | Problem dimension | 1   | 1   | 1   | 7589                              | 140                     | 7589                           | 65.32                   | 0%      |
| Medium dimension  | PR3          | Problem dimension | 2   | 2   | 1   | 3520                              | 1330                    | 3520                           | 179.827                 | 0%      |
|                   | PR4          | Problem dimension | 2   | 2   | 1   | 3965                              | 2850                    | 2932                           | 263.8                   | 2%      |
|                   | PR5          | Problem dimension | 2   | 3   | 2   | 4187                              | 3360                    | 4062                           | 394.9                   | 0.02%   |
| Large dimension   | PR6          | Problem dimension | 3   | 4   | 2   | -                                 | -                       | 5564                           | 567.4                   | -       |
|                   | PR7          | 7                 | 3   | 4   | 2   | -                                 | -                       | 6036                           | 664.7                   | -       |

Therefore, given that the difference in the results obtained from the mathematical model and the metaheuristic method is minor and negligible, and considering the nature of the selected problem, the genetic metaheuristic algorithm provides a reasonable solution that is consistent with the proposed model.

#### 4-5) Sensitivity Analysis

In this section, the sensitivity of the parameters on the emergency vehicle routing models and multi-objective multiple allocation is presented. First, for the values of  $\epsilon$ , the value of the objective functions in the emergency vehicle routing model will be measured and the results obtained, including reliable values, will be reported by determining the distance  $\epsilon$  for the objective functions. For this purpose, in Table 21, different values for  $\epsilon$  are defined and the objective functions are solved by them. As observed in Table 21, the values of the objective functions do not show a significant change with an increase in  $\epsilon$  up to a certain value. However, beyond a certain point (for example, for the second objective function), an increase in  $\epsilon$  leads to a significant upward trend in the values of the objective functions (as  $\epsilon$  changes from 600 to 900). Based on the results obtained, for testing different values of epsilon, the feasible region and the improvement vector of the target functions have been created. According to the results obtained, the level of significant changes of epsilon between 50 and 900 has been determined as the improvement operator. Identifying this interval determines that if the value of epsilon is considered less than 50 and more than 900, the solution to the problem is outside the feasible region. Therefore, the changes range of epsilon for seeking the local optimal solution for the first objective function is 650, as the optimal solution for the first objective function occurs at this point. The optimal state for the second objective function is obtained at epsilon 600. Therefore, if epsilon is selected between 600 and 650, unbeatable solutions are obtained for the problem; otherwise, they are considered unbeatable solutions. Table 21 shows the results of solving the model with a step length of 50.

**Table 21.** Results of Solving the Model with the E-Constraint Method

| $\varepsilon$ | Cost (currency) | Repair time (minutes) |
|---------------|-----------------|-----------------------|
| 50            | 1512            | 57                    |
| 100           | 1563            | 55                    |
| 150           | 1582            | 62                    |
| 200           | 1550            | 57                    |
| 250           | 1530            | 68                    |
| 300           | 1513            | 75                    |
| 350           | 1598            | 58                    |
| 400           | 1565            | 54                    |
| 450           | 1548            | 56                    |
| 500           | 1507            | 58                    |
| 550           | 1580            | 52                    |
| 600           | 1512            | 50                    |
| 650           | 1505            | 72                    |
| 700           | 1550            | 98                    |
| 750           | 1560            | 78                    |
| 800           | 1513            | 81                    |
| 850           | 1515            | 82                    |
| 900           | 1598            | 70                    |
| Optimal value | 1505            | 50                    |

## 5) Conclusion

This study proposes a multi-objective optimization model for optimal route allocation of relief vehicles to improve humanitarian logistics and control costs. The objective functions include minimizing the total cost and vehicle repair time. The epsilon constraint method was used to solve the mathematical model, which is able to determine the dominant and non-dominated solutions without subjective judgments. The application of the model in a case study for optimal route allocation in response to crises shows that the results obtained perform better than current schemes. Additionally, an integer model is presented for the allocation of essential items to different destinations, reducing the cost and number of possible routes. The results show that the shortest routes are determined for the allocation of essential items and the total allocation cost is calculated to be 1,492 monetary units. Finally, a comparison between the proposed model and the NSGA-II meta-heuristic algorithm is conducted, which confirms the efficiency and effectiveness of the model and provides guidance for improving decision-making in humanitarian procurement. Since improving the information and communication management system is identified as a factor of great importance in the successful implementation of the humanitarian supply chain in crisis response, it is suggested that,

when facing natural disasters, the responsible organizations should announce accurate information from the early stages of the incident. This would enable spontaneous organizations and NGOs to collaborate with the responsible organizations in providing relief items. Decision-making about the route of facilities in response to disasters is usually based on the experience of decision-makers or ad hoc decision-making. Therefore, the proposed model improves both the efficiency and effectiveness of humanitarian relief procurement. The findings provide guidance for improving decision-making for route allocation in the field of humanitarian procurement and will be useful for designing an appropriate disaster response strategy in the future. For future research, it is suggested that a multi-period model be developed for multiple complete coverage location.

### Statements and Declarations

**Ethics Statement:** The author affirms that this research was carried out in compliance with recognized ethical standards and principles. All information sources have been appropriately cited and credited. Where required, ethical clearance was granted, and informed consent was obtained from all participants. No instances of plagiarism, data fabrication, or falsification were involved in this study.

**Use of Artificial Intelligence:** The author confirms that AI-based tools were employed exclusively for language polishing and grammatical correction. All AI-generated assistance was thoroughly examined, validated, and revised by the author, who bears full responsibility for the precision, authenticity, and integrity of the final manuscript. AI tools were not utilized to produce or alter research data or to substitute essential scholarly reasoning.

**Conflicts of Interest:** Any possible conflicts of interest, including financial or personal relationships that might affect the research, are disclosed. The author states that there are no competing interests.

**Compliance with Regulations:** The author verifies that this study adhered to all relevant institutional, national, and international rules, and that all required permissions for data usage and reproduction were secured.

### References

- Abdolhossein Zadeh, S., Zandieh, M., & Alam-Tabriz, A. (2025). E-Healthcare improvement through the design of a cloud-based blood supply chain: A system dynamics approach. *Journal of Engineering Management and Soft Computing*, 11(1), 47-1. <https://doi.org/10.22091/jemsc.2025.11210.1201>
- Abolghasemian, M., Bigdeli, H., & Shamami, N. (2024). Locating Routing Problem (LRP) of distribution of priority support items to ground forces in war conditions. *Engineering Management and Soft Computing*, 10(1), 262-292. <https://doi.org/10.22091/jemsc.2023.9453.1232>
- Abolghasemian, M., Ghane Kanafi, A., & Daneshmandmehr, M. (2020). A two-phase simulation-based optimization of hauling system in open-pit mine. *Interdisciplinary Journal of Management Studies*, 13(4), 705-732. <https://doi.org/10.22059/ijms.2020.283100.673669>
- Anaya-Arenas, A. M., Ruiz, A., & Renaud, J. (2018). Importance of fairness in humanitarian relief distribution. *Production Planning & Control*, 29(14), 1145-1157. <https://doi.org/10.1080/09537287.2018.1518557>
- Cao, C., Liu, Y., Tang, O., & Gao, X. (2021). A fuzzy bi-level optimization model for multi-period post-disaster relief distribution in sustainable humanitarian supply chains. *International Journal of Production Economics*, 235, 108081. <https://doi.org/10.1016/j.ijpe.2021.108081>
- Cheng, C., Zhu, R., Costa, A. M., Thompson, R. G., & Huang, X. (2021). Multi-period two-echelon location routing problem for disaster waste clean-up. *Transportmetrica A: Transport Science*, 18(3), 1335-1365. <https://doi.org/10.1080/23249935.2021.1916747>
- Du, J., Wang, X., Wu, X., Zhou, F., & Zhou, L. (2022). Multi-objective optimization for two-echelon joint delivery location routing problem considering carbon emission under online shopping. *Transportation Letters*, 15(7), 652-670. <https://doi.org/10.1080/19427867.2022.2096781>
- Fallahtafti, A., Ardjmand, E., Young II, W. A., & Weckman, G. R. (2021). A multi-objective two-echelon location-routing problem for cash logistics: A metaheuristic approach. *Applied Soft Computing*, 111, 107685. <https://doi.org/10.1016/j.asoc.2021.107685>

- Fathi, M. R., Saffarinia, A., & Pahlevanzadeh, M. J. (2026). Multi-objective green supply chain network design considering carbon emissions and electricity consumption. *Journal of Engineering Management and Soft Computing*, 12(1), 14-35. <https://doi.org/10.22091/jemsc.2026.14628.1326>
- Gandra, V. M. S., Çalık, H., Wauters, T., Toffolo, T. A., Carvalho, M. A. M., & Berghe, G. V. (2021). The impact of loading restrictions on the two-echelon location routing problem. *Computers & Industrial Engineering*, 160, 107609. <https://doi.org/10.1016/j.cie.2021.107609>
- Giedelmann-L, N., Guerrero, W. J., & Solano-Charis, E. L. (2022). System dynamics approach for food inventory policy assessment in a humanitarian supply chain. *International Journal of Disaster Risk Reduction*, 81, 103286. <https://doi.org/10.1016/j.ijdr.2022.103286>
- Gorgi, M., Kaveh, K., Aliakbarian, N., & Ejtehadi, M. R. (2025). Spatio-temporal Moran dynamics in continuous media. *arXiv preprint arXiv:2512.14171*. <https://doi.org/10.48550/arXiv.2512.14171>
- Hasanpour Jesri, Z. S., Eshghi, K., Rafiee, M., & Van Woensel, T. (2022). The multi-depot traveling purchaser problem with shared resources. *Sustainability*, 14(16), 10190. <https://doi.org/10.3390/su141610190>
- Heidari, A., Imani, D. M., Khalilzadeh, M., & Sarbazvatan, M. (2022). Green two-echelon closed and open location-routing problem: Application of NSGA-II and MOGWO metaheuristic approaches. *Environment, Development and Sustainability*, 25, 11467–11503. <https://doi.org/10.1007/s10668-022-02534-1>
- Izadi, E., Nikbakht, M., Feylizadeh, M., & Shahin, A. (2025). Ranking of criteria affecting humanitarian supply chain services based on blockchain platforms using multi-criteria decision-making methods. *Journal of Engineering Management and Soft Computing*, 10(2), 143-160. <https://doi.org/10.22091/jemsc.2025.11552.1215>
- Jahangiri, S., Abolghasemian, M., Pourghader Chobar, A., Nadaffard, A., & Mottaghi, V. (2021). Ranking of key resources in the humanitarian supply chain in the emergency department of Iranian hospital: A real case study in COVID-19 conditions. *Journal of Applied Research on Industrial Engineering*, 8(5), 1-10. <https://doi.org/10.22105/jarie.2021.275255.1263>
- Jahangiri, Sh., Abolghasemian, M., Ghasemi, P., & Pourghader Chobar, A. (2021). Simulation-based optimization: analysis of the emergency department resources under COVID-19 conditions. *International Journal of Industrial and Systems Engineering*, 39(4), 461-481. <https://doi.org/10.1504/IJISE.2021.120358>
- Khan, S. A., & Ali, M. (2023). A robust multi-objective location-routing design for emergency medical supplies under deep uncertainty. *International Journal of Disaster Risk Reduction*, 92, 103745. <https://doi.org/10.1016/j.ijdr.2023.103745>
- Mamashli, Z., Bozorgi-Amiri, A., Dadashpour, I., Nayeri, S., & Heydari, J. (2021). A heuristic-based multi-choice goal programming for the stochastic sustainable-resilient routing-allocation problem in relief logistics. *Neural Computing and Applications*, 34, 15473–15499. <https://doi.org/10.1007/s00521-021-06703-9>
- Mohamed, I. B., Klibi, W., Sadykov, R., Şen, H., & Vanderbeck, F. (2022). The two-echelon stochastic multi-period capacitated location-routing problem. *European Journal of Operational Research*, 303(2), 601-622. <https://doi.org/10.1016/j.ejor.2022.02.046>
- Momen-Vaghefi, S. M., Siahooei, A. A., Sohrabi, P., Andjomshoaa, A., Khawary, H., & Lomer, A. R. B. (2026). Spatial Analysis of Urban Quality of Life in Tabriz City by Comparing Multi-Criteria Decision-Making Models and Different Perspectives. *Sciences of Conservation and Archaeology*, 38(2), 577-596. <https://doi.org/10.48141/sci-arch-38.2.26.46>
- Narimani, R., Motamedi, M., & Khalili, H. A. (2024). A mathematical programming model for the distribution of relief items in a humanitarian supply chain. *Emergency Management*, 13(1), 51-69. <https://doi.org/10.22034/em.2024.19854.1082>
- Nozari, H., Tavakkoli-Moghaddam, R., & Gharemani-Nahr, J. (2022). A neutrosophic fuzzy programming method to solve a multi-depot vehicle routing model under uncertainty during the COVID-19 pandemic. *International Journal of Engineering*, 35(2), 360-371. <https://doi.org/10.5829/ije.2022.35.02b.11>
- Peng, D., Ye, C., & Wan, M. (2022). A multi-objective improved novel discrete particle swarm optimization for emergency resource center location problem. *Engineering Applications of Artificial Intelligence*, 111, 104725. <https://doi.org/10.1016/j.engappai.2022.104725>

- Rezaei Kallaj, M., Abolghasemian, M., Moradi Pirbalouti, S., Sabk Ara, M., & Pourghader Chobar, A. (2021). Vehicle routing problem in relief supply under a crisis condition considering blood types. *Mathematical Problems in Engineering*, 2021, 7217182. <https://doi.org/10.1155/2021/7217182>
- Shao-hong, Y., Jia-yang, N., Tai-long, C., Qiu-tong, L., Cen, Y., Jia-qing, C., Zhi-zhen, F., & Jie, L. (2022). Location algorithm of transfer stations based on density peak and outlier detection. *Applied Intelligence*, 52, 14321–14333. <https://doi.org/10.1007/s10489-022-03181-w>
- Silva, R. F., Santos, B. F., & Mota, C. M. (2024). Ensuring equity in humanitarian logistics: A stochastic multi-echelon vehicle routing model under demand fluctuations. *European Journal of Operational Research*, 315(2), 542-558. <https://doi.org/10.1016/j.ejor.2023.11.039>
- Sohrabi, P., Taheri, J., Nasiri, B., Borna, M., & Soltannia, R. (2026). Assessment of the Quality of Life in Tehran: A Comparison of Multi-Criteria Decision-Making Methods Using GIS. *Sciences of Conservation and Archaeology*, 38(2), 259-274. <https://doi.org/10.48141/sci-arch-38.2.26.46>
- Wang, B. C., Qian, Q. Y., Gao, J. J., Tan, Z. Y., & Zhou, Y. (2021). The optimization of warehouse location and resources distribution for emergency rescue under uncertainty. *Advanced Engineering Informatics*, 48, 101278. <https://doi.org/10.1016/j.aei.2021.101278>
- Wang, L., & Zhang, Y. (2025). Fleet repositioning and breakdown management in dynamic humanitarian supply chains: A machine-learning-assisted metaheuristic approach. *Transportation Research Part E: Logistics and Transportation Review*, 184, 103412. <https://doi.org/10.1016/j.tre.2024.103412>
- Wang, Y., Sun, Y., Guan, X., Fan, J., Xu, M., & Wang, H. (2021). Two-echelon multi-period location routing problem with shared transportation resource. *Knowledge-Based Systems*, 226, 107168. <https://doi.org/10.1016/j.knosys.2021.107168>
- Wei, X., Qiu, H., Wang, D., Duan, J., Wang, Y., & Cheng, T. C. E. (2020). An integrated location-routing problem with post-disaster relief distribution. *Computers & Industrial Engineering*, 147, 106632. <https://doi.org/10.1016/j.cie.2020.106632>
- Yofrido, F. M., & Harjana, L. T. (2019). Social-fairness perception in natural disaster, learn from Lombok: A phenomenological report. *Indonesian Journal of Anesthesiology and Reanimation*, 1(1), 1-7. <https://doi.org/10.20473/ijar.V1I12019.1-7>
- Yu, X., Zhou, Y., & Liu, X. F. (2020). The two-echelon multi-objective location routing problem inspired by realistic waste collection applications: The composable model and a metaheuristic algorithm. *Applied Soft Computing*, 94, 106477. <https://doi.org/10.1016/j.asoc.2020.106477>