




Data Envelopment Analysis for Modeling Efficiency in the Deployment of Military Units for Humanitarian Missions

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Article Info	ABSTRACT
<p>Article type: Research Article</p> <p>Article history: Received 7 February 2026 Received in revised form 1 April 2026 Accepted 10 June 2026 Published online 1 July 2026</p> <p>Keywords: Data envelopment analysis, bootstrap, humanitarian supply chain.</p>	<p>Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the efficiency of decision-making units with similar functions operating under comparable conditions. In humanitarian missions, particularly during crises, identifying efficient patterns for deploying military units is critical to the speed and effectiveness of rescue operations. However, uncertainty in environmental conditions and field information can reduce the accuracy of efficiency measurement. This research proposes a DEA-based framework to evaluate and optimize the deployment of military units in humanitarian operations using bootstrap simulation. A three-stage DEA approach combined with a bootstrap method, grounded in natural, managerial, and free accessibility principles, is applied to data collected from active operational units in a real-world crisis response. A bootstrap based multi stage DEA model was applied to evaluate 16 units. The results showed that only one unit (6.25%) was efficient across all stages, while 93.75% of the units were identified as inefficient, indicating stricter efficiency discrimination after bootstrap adjustment. Results indicate that only a subset of units is efficient under deterministic data, while many are classified as locationally inefficient. After generating simulated data and removing environmental noise, efficiencies are recalculated and comparative changes in unit performance are observed. These findings support more reliable decision-making and provide practical guidance for planners seeking robust, data-driven deployment strategies under uncertainty in complex humanitarian crisis environments.</p>
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1) Introduction

Many disasters are unpredictable, causing severe casualties, financial losses, environmental damage, and serious social harm, or potentially leading to them (Wang et al., 2021). Decision-making, regarding the allocation of shelters, significantly impacts the success of disaster response and ensures the necessary security for victims. For example, the crisis that recently affected the world, the emergence of COVID-19 in China at the end of 2019, placed the world in a severe crisis, resulting in more deaths than any year in previous decades (Jahangiri et al., 2021). Additionally, according to statistical data on natural disasters reported by the Epidemiology Research Center, natural disasters such as earthquakes and floods, alongside all human-made crises, have been the most common disasters, resulting in 5,110 fatalities (Abolghasemian et al., 2020). Among these, humanitarian logistics plays a crucial role in facilitating disaster management processes, evacuating victims from affected areas to safe locations, and planning, storing, and distributing aid equipment to assist victims in a timely, appropriate location, and at a reasonable cost (Jahangiri et al., 2023). However, limited resources create a significant problem in the humanitarian supply chain. During widespread crises, it is unfair to prioritize one region and allow it to receive the entire demand for its services. At the same time, another region may receive less than its fair share due to supply shortages (Yofrido & Harjana, 2019). Therefore, ethical decisions are challenging for decision-makers. However, when relief packages are insufficient to meet all demands, such conditions require an appropriate allocation approach that can distribute aid items to affected areas in a way that considers equal priority for meeting the needs of different segments of the affected population (Poornaser et al., 2022). Given that different areas have different needs, fair behavior is defined as the approach to achieving demand levels or dealing with demand shortages. Therefore, using the humanitarian supply chain after a disaster, emergency logistics is necessary to distribute relief goods fairly across all crisis points (Zheng et al., 2015). In this case, relief items or goods should be allocated to affected points in proportion to their demand. In other words, when relief resources are constrained, it may be better to meet some of the needs across affected areas than to fully satisfy one demand point while entirely neglecting another (Anaya et al., 2018).

Natural and human disasters such as floods, earthquakes, and volcanic eruptions demonstrate a rising trend in human societies. The extent, severity, and effects of disasters are such that they cause significant damage to human lives and property, which in turn has led to significant global attention towards addressing these disasters. Supply chain management includes all activities involved in connecting suppliers, manufacturers, distributors, and customers to produce and distribute goods at an appropriate quantity, at the right time, at minimum system costs, and with maximizing customer service levels. The humanitarian supply chain is a specific type of supply chain with unique characteristics that distinguish it from commercial supply chains. The performance of this supply chain plays a very important role in controlling and reducing the effects of disasters. Humanitarian operations are conducted to preserve lives and reduce the suffering of people in crises. These operations include the preparation of materials and technical aid along with the provision of essential services in crisis situations where the ability of communities to provide these items is severely limited. Humanitarian operations are not aimed at profit. Humanitarian operations are temporary in nature and are carried out with the goal of the recovery and self-sufficiency of affected communities (Jahangiri et al., 2021). According to the definition of the World Health Organization (WHO), a crisis is an event that results in destruction, damage, environmental disruption, human death or injury, or a decline in health or health services within a population group, such that it requires assistance from communities outside the affected area.

In other words, the occurrence of a crisis is a sudden breakdown of the connection and continuity between humans and the environment (Abolghasemian et al., 2021). Every year, more than 500 crises threaten our planet, resulting in the death of 75,000 people and affecting more than 200 million others. In most crises, there is a massive demand that exceeds available resources. This demand exceeding resources causes difficulties in preparing and supplying relief materials and resources for affected individuals, ultimately affecting the complete success of the planning, management, and control processes of these resources for providing assistance to victims. Behind the success of every

humanitarian aid and relief program, there are logistical processes and supply chains (Eligüzel et al., 2023). This supply chain must respond to unexpected events and be structured to meet specific and separate success needs. Among all areas of the humanitarian supply chain, performance evaluation has special importance and received less attention than other areas. However, among organizations active in the humanitarian field, only 45% have addressed the issue of performance evaluation, and among these, only 20% continuously evaluate the performance of all their activities (Rezaei Kallaj et al., 2021). Therefore, by providing effective performance measurement, relief performance improves and transparency in accountability in relief operations enhances. In this case, relief organizations gain a proper understanding of their humanitarian supply chain through evaluating their supply chain performance in past disasters and obtain appropriate feedback from the information obtained from evaluating their supply chain for improving the performance of supply chains in future disasters (Cao et al., 2024).

In crisis situations, such as natural disasters or military conflicts, the need for humanitarian operations to provide immediate assistance to victims and save human lives is strongly felt. In this regard, military units, as key institutions in ensuring security and facilitating humanitarian operations, must be effectively located to be able to respond to human needs as effectively as possible. Given the complexities in crisis conditions and the need for rapid and effective responses, optimizing the location of military units has become a fundamental challenge. Data Envelopment Analysis (DEA) is a technique specifically designed to examine the efficiency of units under evaluation. A unit under evaluation refers to one where a certain number of outputs are produced using a certain number of inputs. Efficiency expresses how an organization uses its best performance at a point in time. The concept of efficiency is often confused with the terms effectiveness and productivity. Effectiveness expresses the degree of alignment of an organization's activities with its defined objectives. However, productivity is a combination of efficiency and effectiveness. There are two points of difference between the concepts of efficiency and productivity. First, efficiency is a measure on a scale of zero to one and is expressed as a percentage, while productivity can be greater than one. Second, productivity is calculated in relation to individual factors such as labor or capital productivity, but efficiency, as a general measure of the combination of inputs and outputs, must be obtained. Methods for measuring efficiency are generally divided into parametric and non-parametric categories. In parametric methods, estimates are made using various statistical and econometric methods, and then efficiency is determined using this function. However, in non-parametric methods, there is no need to estimate the production function. DEA is one of the non-parametric methods for measuring the efficiency of units. DEA is a linear programming-based method that has the ability to measure the relative efficiency of units with multiple similar inputs and outputs. The DEA method was initially introduced for measuring efficiency, but after a while, it attracted the attention of experts in the field of multi-criteria decision-making and became accepted as one of the multi-criteria decision-making techniques; therefore, the evaluated options are assumed to be one DMU, and the relevant DEA models are solved by considering negative criteria as inputs and positive criteria as outputs of decision-making units.

In times of crisis, the performance of the humanitarian supply chain plays a vital role in ensuring the timely and effective delivery of essential services. Due to their organized logistics systems, rapid response capability, and access to critical operational resources, military units often serve as key actors supporting humanitarian operations and bridging gaps in emergency response (Wang et al., 2021). Evaluating the efficiency of these units within the humanitarian supply chain requires a robust analytical framework capable of capturing the multi-stage nature of relief activities and the multiple inputs and outputs involved. The DEA method, as a non-parametric performance measurement approach, provides an appropriate methodological foundation for assessing the relative efficiency of such operational units, particularly when dealing with complex processes and limited or noisy data. Accordingly, integrating the humanitarian supply chain context with the operational role of military units and the analytical capabilities of DEA enables a comprehensive understanding of their effectiveness during crises and highlights areas where performance improvement is feasible (Poornaser et al., 2022).

Location is the process of analyzing the capabilities and abilities of a region regarding the availability of suitable and sufficient land and its connection with other uses to select an appropriate

location for a specific use. Location of services not only affects costs but also influences their efficiency, utilization, and quality. Discrete location problems include determining the location of one or more facilities in one or more potential locations to minimize the cost of meeting customer needs. Due to the limited capacity of facilities, the allocation problem often needs to be solved in conjunction with the location problem. Location-allocation models simultaneously seek to determine the optimal location of facilities and allocate customers to selected facilities. In other words, the location-allocation problem involves the deployment of a set of new facilities in such a way that the transportation cost from facilities to customers is minimized and an optimal number of facilities are placed in the area of interest to meet customer demand. Objectives, constraints, and variables of location problems vary depending on the purpose of the location and the type of problem under investigation.

There are two main approaches for applying the concept of efficiency in location problems. In the first approach, the location problem under consideration is viewed as a discrete multi-criteria decision-making problem, where the objective is to select the most efficient location or locations from among the existing candidate points for a specified number of facilities. The second approach uses mathematical location models to determine suitable locations. These location models are integrated hierarchically or simultaneously into DEA models and lead to the determination of efficient locations.

Based on the above points, the objective of this research is to develop an optimization model for the location of military units in field humanitarian operations, which, by using DEA, helps to identify and evaluate the efficiency of different location options. This model can assist decision-makers in selecting the best options for deploying military units by considering various criteria, thereby having a greater impact on providing humanitarian assistance. The present study addresses a gap in the disaster management and humanitarian logistics literature regarding performance-based evaluation and location planning of military units. The main innovation of this research is the development of an optimization model for locating military units in field humanitarian operations using DEA. The proposed model simultaneously considers multiple operational inputs and outputs to evaluate the efficiency of alternative deployment locations. By integrating an efficiency-based DEA approach with the problem of military unit location in humanitarian supply chains, the model enables the comparison of different location options and the identification of more efficient deployment patterns. Consequently, it provides a structured decision-support tool for improving the effectiveness and responsiveness of humanitarian relief operations in crisis situations.

The rest of paper is organized as follows: first, a review of the literature on crisis management and resource allocation is presented. Then, the DEA model and the proposed framework for deploying military units in humanitarian missions are described. Next, the case study and the data used are introduced, and the model results are analyzed. Finally, a discussion, conclusion, and directions for future research are provided.

2) Literature Review

Determining the location of facilities for disaster response is a key topic in humanitarian logistics decision-making. In previous studies, single-objective optimization models and multi-objective computational methods have frequently been used to improve the efficiency of relief supplies, with facilities such as shelters, medical centers, warehouses, and distribution centers considered (Boonmee et al., 2017). Some research also relies on the subjective judgment of decision-makers (Roh et al., 2018). A review of the literature indicates that most models have focused on optimizing profitability or non-profitability, using methods such as weighted goal programming and weighted objective functions to solve multi-objective problems; these methods can lead to errors in prioritizing criteria in humanitarian logistics. Examples of such studies include single-objective models for reducing the number of shelters (Ozbay et al., 2019), minimizing cost (Praneetpholkrang et al., 2020), and maximizing decision-maker satisfaction (Kanoun et al., 2010). In recent years, more extensive research has been conducted in this field. For instance, a multi-objective location model under uncertainty (Maharjan & Hanaoka, 2019), the identification of success factors in humanitarian supply chains (Motamedi et al., 2019), multi-modal distribution models (Maghfiroh & Hanaoka, 2020), and models for reducing casualties and unmet demand (Mansoori et al., 2020) have been proposed. Research by Sabouhi et al. (2020) also developed

two-stage stochastic models for location and routing. Subsequently, multi-objective scenario-based models and robust fuzzy approaches for resource allocation and routing in crisis situations have been introduced (Mamashli et al., 2021). Other studies in this field include evaluation of hospital supply chain elements using multi-criteria decision-making methods (Jahangiri et al., 2021), blood supply routing in earthquakes (Rezaei Kallaj et al., 2021), as well as scheduling and routing of organ transplant ambulances and emergency warehouse location under uncertainty (Wang et al., 2021). More recent research includes two-level, three-objective models for minimizing unmet demand, cost, and environmental risk (Cao et al., 2021), emergency center location with multi-objective models and particle swarm algorithms (Peng et al., 2022), improvement programs for operations during a pandemic (Poornaser et al., 2022), multi-level models for distributing relief packages (Narimani et al., 2023), and multi-objective relief models emphasizing social dissatisfaction. Recent studies have also focused on optimizing medical waste location (Cao et al., 2024).

Despite the growing body of research on humanitarian logistics and crisis management, several important gaps remain in the existing literature. First, although numerous studies have addressed facility location and resource allocation in disaster response, most of them primarily focus on cost or response time minimization and pay limited attention to the comprehensive evaluation of operational performance. Second, while DEA has been widely applied for efficiency assessment in various sectors, its application in evaluating the performance of actors involved in humanitarian logistics, particularly military units that often play a critical logistical role in disaster relief, remains limited. In addition, many existing approaches do not adequately capture the multi-input and multi-output nature of relief operations or the complex structure of humanitarian supply chains. To address these gaps, this study develops an analytical framework for evaluating the efficiency of military units involved in humanitarian relief operations using DEA within the context of humanitarian supply chains. By integrating efficiency evaluation with the operational characteristics of disaster response systems, the proposed approach provides a more comprehensive perspective for assessing the performance of relief units and supporting decision-making in emergency logistics. The findings of this research contribute to the literature on humanitarian logistics by offering a systematic framework for performance evaluation and by highlighting the role of military logistical capabilities in improving disaster response effectiveness.

3) Methodology

The main idea of the method for estimating the performance and sustainable products through bootstrap simulation is to numerically simulate real data and calculate the efficiency of the simulated data, inspired by the study of Sink et al. (2019). In this study, unlike Singh et al. (2019), which performed bootstrap simulation using the traditional CCR data envelopment analysis (DEA) model, a DEA model based on the combination of natural, managerial, and free accessibility principles has been proposed. For details of the methodology, the following steps are considered.

Step 1: For each DMU that has x_k inputs and y_k outputs, we obtain the efficiency score $\widehat{\theta}_k$ by using a suitable DEA model according to the nature and main structural form of the system.

Step 2: Using the value of $\widehat{\theta}_k$ for each DMU using bootstrap, the random efficiency score $\theta_{1b}^*, \dots, \theta_{nb}^*$ is calculated.

Step 3: We calculate the simulation value for (X_{kb}, Y_{kb}) according to equation 4:

$$X_{kb} = \left(\widehat{\theta}_k / \theta_{nb}^* \right) \cdot X_k, \quad Y_{kb} = \left(\widehat{\theta}_k / \theta_{nb}^* \right) \cdot Y_k \quad (4)$$

Step 4: For each simulated sample, we must again use the DEA method to calculate the efficiency score θ_{bk}^* .

By repeating the second to fourth steps to B times, we will have a set of estimated score θ_{bk}^* .

3-1- Proposed Framework for Efficiency Analysis Based on Principles of Natural and Managerial Accessibility

The main idea of the method for estimating performance evaluation through bootstrap simulation is to numerically simulate real data and calculate the efficiency of the simulated data, inspired by the study

of Sink et al. (2019). Since the proposed model is based on the two principles of natural and managerial accessibility, we consider the following steps for details of the proposed model based on these two principles:

Principle of Natural Accessibility: According to the principle of natural accessibility, a DMU reduces its input vector to increase its output vector. In this case, by reducing inputs, a DMU can increase its desirable outputs as much as possible. This principle is actually known as a negative correlation. According to this principle, there is a trade-off between economic development and environmental factors. In this type of accessibility, a DMU attempts to achieve an efficient frontier for desirable outputs and inputs when measuring efficiency.

Principle of Managerial Accessibility: According to this principle, unlike natural accessibility, it shows that a DMU increases or maintains its input vector at the current level to increase desirable outputs and reduce undesirable ones. Therefore, according to this principle, by increasing or maintaining inputs in a DMU, we increase desirable outputs as much as possible under new technology or new management. In this research, the use of managerial accessibility is considered as a capability that has positive adaptability to changes in operational conditions. Managerial accessibility often leads to increased desirable outputs and system performance maintenance through investment in existing facilities, which is supported by some organizations. In this research, we are essentially aiming to evaluate efficiency by directly applying the simultaneous effects of both natural and managerial accessibility principles through the construction of new technology.

In this research, the selection of input and output indicators in the DEA model is based on an in-depth literature review of humanitarian logistics and sustainable supply chain management. The indicators reported in Table 1 were chosen from among the most frequently used and empirically validated factors in prior studies; therefore, they jointly reflect the three main pillars of sustainability economic, environmental, and social performance across the deployment, response, and rescue stages of the humanitarian supply chain. From an economic perspective, indicators such as Unit Deployment Cost and Resource Consumption are modeled as inputs (or cost-type criteria), as they represent the resources that must be minimized for a given level of mission performance. Environmental indicators, such as Reduction in Environmental Damage and CO₂ Emission, capture the trade-off between operational efficiency and environmental impact under the principle of natural accessibility, where units are expected to reduce resource use and pollution while maintaining or improving outputs. Social indicators, such as Response Time to Affected Areas and Local Community Satisfaction, are treated as desirable outputs or linking variables, since they directly represent the effectiveness and quality of humanitarian services delivered to beneficiaries. In line with the principles of natural and managerial accessibility, the chosen inputs and outputs allow the DEA model to account for both (i) improvements achieved by reducing inputs for a given level of outputs, and (ii) improvements achieved by maintaining or increasing inputs in order to increase desirable outputs and reduce undesirable ones under new technologies or management practices. Therefore, the configuration of inputs and outputs in Table 1 is consistent with both the theoretical foundations of DEA and the operational characteristics of sustainable humanitarian missions. Table 1 indicates the inputs and outputs of the proposed model for constructing the proposed technology.

Table 1. Introduction of Sustainability Indicators in the Proposed Humanitarian Supply Chain

Sustainability Aspect	Indicator	
Economic	Unit deployment cost	Input
Social	Response time to affected areas	Input
	Local community satisfaction	Middle
Environmental	Reduction in environmental damage	Middle
	CO ₂ emission	Output
	Resource consumption	Input

In the first step, we initiate by evaluating the efficiency of each Decision-Making Unit (DMU) with X_k inputs and Y_k outputs. We achieve this by employing a suitable DEA model, considering the system's inherent characteristics and primary structural form. Our assumption in this research is that by maintaining specific managerial inputs constant (contrary to traditional assumptions), we can enhance the desired outputs. Additionally, we minimize uncontrollable inputs, adhering to the natural principle in DEA. To analyze the data envelopment in our research, we examine J DMUs under evaluation. Each DMU, denoted as DMU_j with $j = 1, \dots, J$, consumes m inputs $X_{mj} = x_{1j}, \dots, x_{mj}$ to produce r desired outputs $Y_{rj} = y_{1j}, \dots, y_{mj}$. Following Sueyoshi and Goto's (2010) classification, inputs are categorized into manageable $x_{qj}^M (q = 1, \dots, Q)$ and unmanageable $x_{pj}^N (p = 1, \dots, P)$ types. Manageable inputs can be improved to eliminate undesirable outputs and enhance desired outputs while maintaining a constant level. Unmanageable inputs are reduced based on the natural principle. By increasing desired output, reducing unmanageable inputs, and keeping manageable inputs constant, we can achieve an efficient unit in new technology construction. Table 2 defines general indices, parameters, and variables for this problem.

Table 2. Symbols and Notation

Symbols	Notes
Sets	
P	Set of uncontrollable inputs
Q	Set of manageable inputs
R	Set of desirable outputs
A	Set of undesirable outputs
J	Set of DMUs (units under evaluation)
D	Set of dedicated inputs
K'	Set of intermediate elements from the first stage to the second (input to the second stage)
K''	Set of intermediate elements from the first stage (output from the first stage)
Inputs, Outputs, and intermediaries	
x_{qj}^M	Management input of DMU_j from set Q
x_{pj}^N	Non-management input of DMU_j from set P
y_{rj}	Desired output of type r from set R in DMU_j
$z_{kk'}$	Intermediate element from step k to step k'
y_{wj}	Undesired output of type w from set A in DMU_j
h_{dj}	Dedicated input of the second stage from set D in DMU_j
$p_{k''K}$	Intermediate element from stage k'' to stage K
λ_j	Shadow price of input and output for DMU_j
Θ	Objective function's value (adjustment coefficient for inputs)

In DEA studies, selecting appropriate inputs and outputs is crucial for efficiency evaluation. Incorrect selections may compromise the study's validity. To ensure accuracy in this research, two key factors are considered during variable selection:

1. The feasibility of collecting and accessing data related to the study's variables.
2. Incorporating the insights of experts and university professors in the field of study.

By adhering to these considerations, the new technology for establishing the feasible area is formulated through equations 1 to 11, which collectively determine the overall efficiency of the supply chain:

$$\begin{aligned} \min \theta & & (1) \\ & \text{s. t.} \\ & \text{First stage} \\ & p = 1, \dots, P & (2) \\ & \sum_{j=1}^n \lambda_j^1 x_{pj}^N \leq \theta x_{pk}^N \\ & q = 1, \dots, Q & (3) \\ & \sum_{j=1}^n \lambda_j^1 x_{qj}^M = x_{qk}^M \\ & k = 1, \dots, K & (4) \\ & \sum_{j=1}^n \lambda_j^1 z_{kj} \geq z_{k'k} \\ & \text{Second stage} \\ & a = 1, \dots, A & (5) \\ & \sum_{j=1}^n \lambda_j^2 w_{aj} \leq \theta w_{ak} \\ & k' = 1, \dots, K' & (6) \\ & \sum_{j=1}^n \lambda_j^2 z_{k'j} \leq z_{k'k} \\ & d = 1, \dots, D & (7) \\ & \sum_{j=1}^n \lambda_j^2 h_{dj} \geq h_{dk} \\ & k'' = 1, \dots, K'' & (8) \\ & \sum_{j=1}^n \lambda_j^2 p_{k''j} \geq p_{k''k} \\ & \text{Third Stage} \\ & r = 1, \dots, R & (9) \\ & \sum \lambda_j^3 y_{rj} \geq y_{rk} \\ & k'' = 1, \dots, K'' & (10) \\ & \lambda_j^f \geq 0 & \theta \text{ unrestricted}; f = 1,2,3 & (11) \end{aligned}$$

In equation (1), the objective function is to determine the minimum value of inputs to be on the efficiency frontier. In equation (2), $\sum_{j=1}^n \lambda_j^1 x_{pj}^N \leq \theta x_{pk}^N$ is related to non-unmanageable inputs, stating that unmanageable inputs are reduced according to the principle of natural disposability. Equation (3), $\sum_{j=1}^n \lambda_j^1 x_{qj}^M = x_{qk}^M$ is related to the manageable inputs of managerial disposability. This constraint adds this capability to the problem so that the costs related to managerial factors remain constant under the control of management. This constraint ignores the reduction in the vector of inputs so as to rise the desired yields and simultaneously reduce the undesirable outputs according to the traditional assumption of DEA. In equation (4), $\sum_{j=1}^n \lambda_j^1 z_{kj} \geq z_{k'k}$ states that the middle element $z_{k'k}$ is the yield of the first stage. Equation (5), $\sum_{j=1}^n \lambda_j^2 w_{aj} \leq \theta w_{ak}$, guarantees that the undesirable output in the system is reduced as much as possible; or if there is no ability to reduce it, it will remain at the existing level. Equation (6) $\sum_{j=1}^n \lambda_j^2 z_{k'j} \leq z_{k'k}$ states that the intermediate element $z_{k'k}$ which is the yield of the first stage needs to be considered as the input of the second stage. Equation (7) $\sum_{j=1}^n \lambda_j^2 h_{dj} \geq h_{dk}$ represents the dedicated input of the second stage, which is treated according to the principle of managerial disposability and we try to increase it. Equation (8) $\sum_{j=1}^n \lambda_j^2 p_{k''j} \geq p_{k''k}$ represents the intermediate input $p_{k''j}$ which is the output of the second stage. Equation (9) $\sum \lambda_j y_{rj} \geq y_{rk}$ reflects natural disposability

for desired outputs in the feasible region. Equation (10) $\sum \lambda_j^3 p_{k^*j} \leq p_{k^*k}$ guarantees that p_{k^*k} is considered as the input of the third stage. Equation (11) shows the limits of the considered variables.

Theorem 3-1: Feasibility of Equations (1) through (11)

To establish the feasibility of our proposed model, we must demonstrate that at least one solution exists, where all values of $\lambda_j^1 x_{1j}^M, \lambda_j^1 x_{1j}^N, \lambda_j^2 w_{1j}, \lambda_j^3 y_j$ are feasible.

Proof: We can consider a solution set, such as $\lambda_j^f = 0; \lambda_o^f = 1; \theta = 1$, with $\theta = 1$ for the unit under investigation o . Since this solution applies to all problem adverbs, the problem possesses at least one possible answer.

Theorem 2: In Equations (1) to (10), θ^* is always less than 1.

Lemma (Backward Proof): We assume that the optimal solution is greater than 1.

Proof: We propose that for $\theta = 1$ and all λ s set to zero except for the unit under examination ($\lambda_o^f = 1$), a feasible solution exists. This solution is applicable to all problem constraints and can be represented as $\lambda_j^f = 0; \lambda_o^f = 1; \theta = 1 \forall j \neq o$. As the objective function aims at minimization in each iteration, the value of the objective function decreases, and $\theta^* = 1$ has been calculated. Therefore, the existence of an optimal solution with θ^* greater than one is contradictory, given the constraint $\theta = 1$. This contradiction invalidates the posterior hypothesis, thereby proving that $\theta^* \leq 1$ according to the theorem.

Definition 1: In the context of Equations (1) to (10), if $\theta^* = 1$ and one of the auxiliary variables, $s_i^+ \neq 0$ or s_i^- is non-zero, the model exhibits a weak efficient solution.

Definition 2: In the context of Equations (1) to (10), if $\theta^* = 1$ and all auxiliary variables, $s_i^+ \neq 0$ and s_i^- , are zero, the model demonstrates a strong efficient solution.

4) Findings and Discussion

In this section, a practical example is considered for further analysis and investigation of the introduced approach. For this purpose, a humanitarian relief supply chain is considered based on the considered indicators, with the deployment of nine operational units. This example considers a multi-stage humanitarian supply chain. Figure 1 illustrates a multi-stage humanitarian supply chain model consisting of three sequential phases:

1. **Deployment Phase:** This stage involves the initial setup and mobilization of military units. Key inputs include the cost of deploying units and the time to respond to affected areas. The output of this phase is the reduction in environmental damage due to optimal deployment, which serves as an intermediate variable.
2. **Response Phase:** In this stage, the focus shifts to delivering aid. The intermediate variable from the deployment phase (reduction in environmental damage) becomes an input. Additional inputs include consumption of natural resources. Outputs include CO₂ and pollutant emissions (undesired output) and local community satisfaction with military performance (desired output).
3. **Rescue Phase:** This final stage uses the output from the response phase (community satisfaction) as an input. Other inputs include resource consumption. The outputs consist of total supply chain cost, delivery time, number of affected populations, and environmental impact.

The figure illustrates a clear flow from deployment to response to rescue, with intermediate variables linking the stages, reflecting a comprehensive model for evaluating the efficiency of humanitarian operations.

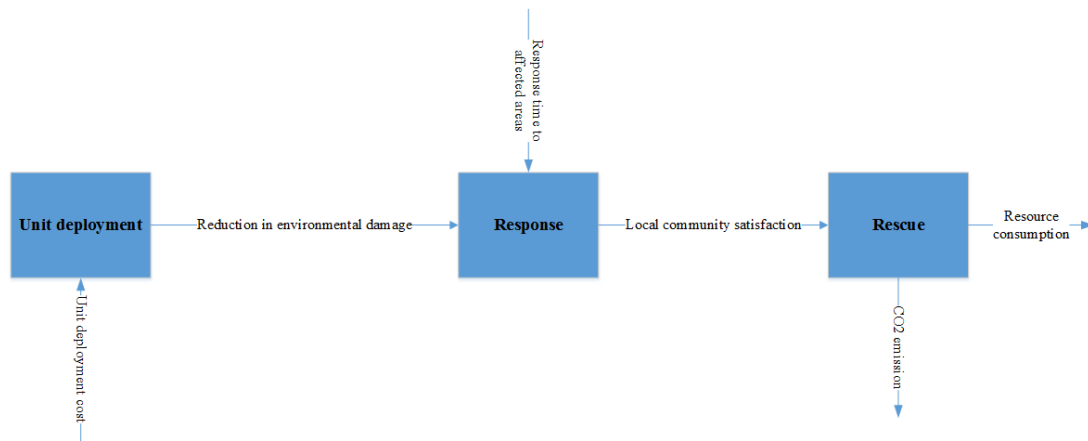


Figure 1. Humanitarian Supply Chain Structure

In Stage (1), Unit Deployment occurs; in Stage (2), Response takes place; and in Stage (3), Rescue is performed. All inputs and outputs of this chain are considered based on an in-depth literature review, following the most frequent factors as outlined in Table 1.

In Stage (1), Unit Deployment Cost is considered a cost criterion for the economic aspect, and Reduction in Environmental Damage is considered the link between Stage (1) and Stage (2) for the environmental aspect. In Stage (2), Response Time to Affected Areas is considered a criterion for social issues. Furthermore, Local Community Satisfaction is considered the link between Stage (2) and Stage (3). In Stage (3), CO₂ Emission is considered the environmental criterion, and Resource Consumption is considered the economic criterion.

We denote the input vector entering Stage (1) (Unit Deployment Cost) as X_1 . Furthermore, Vector Z_{12} (Reduction in Environmental Damage) is the output vector leaving Stage (1) and entering Stage (2) as an input vector. This variable is known as an intermediate variable. Vector X_2 (Response Time to Affected Areas) is the specific input for Stage (2). The undesired output variables are represented by Y_3 as CO₂ emissions released in Stage (2). Vector Z_{23} (Local Community Satisfaction) is the output vector leaving Stage (2) and entering Stage (3) as an input vector. Vector X_3 (Resource Consumption) is considered the specific input for Stage 3. The data is presented in Table (3). Additionally, to align with the principle of managerial accessibility, the Response Time to Affected Areas factor is considered a manageable input, and the Material Deployment Cost is considered an unmanageable factor.

Table 3. Data Set

DMU	Inputs			Middle		Output
	X^1	X^2	X^3	Z_{12}	Z_{23}	Y_3
1	0.83	0.59	0.87	0.62	0.52	0.89
2	0.75	0.73	0.89	0.85	0.65	0.69
3	0.82	0.74	0.78	0.45	0.65	0.78
4	0.92	0.89	0.64	0.68	0.78	0.79
5	0.84	0.74	0.56	0.74	0.74	0.68
6	0.87	0.57	0.78	0.91	0.71	0.86
7	0.73	1	0.56	0.68	0.78	0.78
8	0.94	0.67	0.76	1	0.78	0.69
9	1	0.76	0.81	0.61	0.81	0.72

Therefore, a major advantage of calculating the proposed stage-wise efficiency in this research is that it reveals the reasons for the inefficiency or efficiency of the evaluated units. Other evaluated units are also identified as inefficient decision-making units due to inefficiency in all stages. Finally, a bootstrap efficiency value, calculated as the average of the first, second, and third stage efficiencies, is determined in the last column of Table 4 using the symbol θ_{nb}^* , and is presented to generate data for further application of the proposed framework among the evaluated units.

Table 4. Calculation of Overall Efficiency, Stages, and Bootstrap

DMU	Overall $\hat{\theta}_k$ Efficiency	First Stage Efficiency $\hat{\theta}_{k1}$	Second Stage Efficiency $\hat{\theta}_{k2}$	Third Stage Efficiency $\hat{\theta}_{k3}$	Bootstrap Efficiency θ_{nb}^*
1	0.966	0.668	1.000	0.742	0.844
2	1.000	1.000	1.000	1.000	1.000
3	0.764	0.712	0.744	0.715	0.733
4	1.000	1.000	1.000	1.000	1.000
5	0.984	0.899	1.000	0.742	0.906
6	0.864	0.564	0.864	0.745	0.759
7	0.691	0.860	0.657	0.545	0.688
8	0.977	1.000	0.663	0.782	0.855
9	0.752	0.752	0.598	0.657	0.689

Using the calculated bootstrap efficiency value, we compute the bootstrap random input, output, and intermediate values based on Table 5 to re-calculate the overall and final efficiency scores of the evaluated units. For this purpose, we generate new bootstrap data based on the existing data from a tomato paste producer in Iran. This is because the data generated by bootstrap can eliminate noise in the data and provide better results. Therefore, the efficiency of the evaluated units is recalculated based on the generated data.

Table 5. Bootstrap Simulation Data Set

DMU	Inputs			Middles		Output
	X^1	X^2	X^3	Z_{12}	Z_{23}	Y_3
1	0.91	0.54	0.33	0.28	0.22	0.69
2	0.75	0.73	0.85	0.86	0.62	0.85
3	0.92	0.83	0.37	0.37	0.36	0.79
4	1.03	0.92	0.63	0.52	0.51	0.46
5	0.93	0.69	0.69	0.50	0.48	0.46
6	0.87	0.57	0.91	1	0.95	0.49
7	0.81	1.11	0.75	0.28	0.25	0.49
8	1.75	0.79	0.78	0.8	0.32	0.79
9	1	0.76	0.61	0.86	1	0.65

Using the randomly generated bootstrap data, the overall and stage-wise efficiency of the supply chain has been recalculated. According to the results presented in Table 6, in this case, only DMU2 is identified as the overall efficient unit, being efficient in all three stages, while the other evaluated units are inefficient when using the bootstrap data. Therefore, only 6.25% of the units are efficient. Therefore, by using bootstrap data, a fundamental property of DEA models namely, improved accuracy of efficiency calculations due to data variation and reduced noise, is realized. This is because the number of identified efficient units is lower compared to the non-bootstrap case

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