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Designing a Multi-Objective Mathematical Model for Optimizing a Shop Floor Flow Production System, Considering the Number of Human Activities Using the Gray Wolf Metaheuristic Algorithm

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ABSTRACT

The flow shop production system has attracted significant attention in research, and many researchers have conducted research to optimize flow shop production systems. This is important because in a flow shop system, any number of activities can be performed by any number of machines, and in fact, each machine can perform a variety of activities. The important point is to correctly assign activities to each machine in order to minimize the time to complete all activities. In the present study, a model for optimizing the number of human activities in a flow shop production system is presented, which is based on a two-objective model. Due to the NP hard nature of the problem, metaheuristic algorithms, specifically the multi-objective gray wolf algorithm, were used to solve the model. The result of solving the model shows that the mathematical model has a very good ability to solve the problem in small dimensions. The gap between the results of the deterministic solution and the metaheuristic model has been reported to be zero. With a further increase in the amount of human resources and the increase in the complexity of the problem, while increasing the computational time, the gap between the results also increases to an acceptable level. As far as the human resources are considered equal to 6 to 7, there is no possibility of solving the problem for the proposed model. This point is also true for increasing the number of parts, machines, cycles, and shifts. However, considering that, in small and medium problems, the gap between the mathematical model and the metaheuristic model was negligible, by relying on the results calculated according to the metaheuristic model, the results presented are reliable.

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1) Introduction

There is a consensus among many authors that the creation of initiative in scheduling the necessary activities for product manufacturing is influenced by equipment reliability and maintainability. Product waste generated during production processes is directly proportional to the performance of production equipment (Abolghasemian et al., 2022). Equipment failure leads to low product quality and consequently to delays in delivery and preparation. Therefore, timely fulfillment of customer needs means that equipment availability must be at its peak efficiency and performance (Hemmati et al., 2024). Through systematic and strategic maintenance management, defects and variations caused by poor equipment can be identified and eliminated (Chobar et al., 2022). However, it is observed that the important role of maintenance and repair management, as a crucial component in creating initiative for quality improvement and reducing delays in production planning, is not emphasized in the literature, especially in small and medium-sized industries (Shahidi-Zadeh et al., 2017).

Hybrid flow shop streams are used in many practical production environments, such as papermaking, chemicals, textiles, steel, glass manufacturing, pharmaceuticals, automotive, and electronics industries. In many HFS studies, setup times are assumed to be negligible or can be included in the job processing time. However, scheduling with setup times plays a crucial role in modern manufacturing where high-quality products must be guaranteed and delivered on time. The hybrid flow shop scheduling problem is one of the most widely used production scheduling problems, dating back to the late 1970s. A hybrid flow shop system consists of a series of production stages or workshops, each of which has one or more machines. Some stages may have only one machine, but for such a system to be called a hybrid flow shop, there must be multiple parallel machines in at least one stage (Niavand et al., 2024; Shahidi-Zadeh et al., 2017). The hybrid flow shop problem is a generalized version of two specific types of scheduling problems known as job shop scheduling and parallel machine problems. Therefore, it includes a set of production stages, each with several operational parallel machines. The flow of products in the production system is uniform and unidirectional. Each job is processed by one machine at each stage, and machines at each stage can have different processing speeds. Hence, the goal is not only to determine a sequence for processing jobs in the system but also, in addition to determining the priority and sequence of job processing, to appropriately assign jobs to each of the parallel machines with the aim of achieving an efficiency criterion (Javadi et al., 2024; Kaviyani-Charati et al., 2022). Figure 1 shows a hybrid flow shop system with two machines in the first stage and one machine in the second stage. The problem investigated in this research is the scheduling of a two-stage hybrid flow shop problem, with one machine in the first stage and two parallel machines with different processing times in the second stage. Examples of applications of this problem include two-stage production systems, such as weaving factories, plastic injection, and some industrial production processes like forging, casting, and machining.

The problem considered in this research differs from other hybrid flow shop scheduling problems as there are two dedicated machines in stage 1. These two machines include dedicated machines as one of the production environments for hybrid workshops. Having dedicated machines in stage 1 is also common in real-world situations. For example, depending on product specifications, different types of products are processed on different machines in the production stage. Additionally, they undergo a similar testing and inspection process or a packaging process at the end of final assembly (stage two). The goal of this research is to select an appropriate sequence for processing jobs without restrictions on the number of jobs on the machines in the first stage and to assign each job processed in the first stage to the only available machine in the second stage, so that the defined criteria for evaluating scheduling efficiency are optimized.

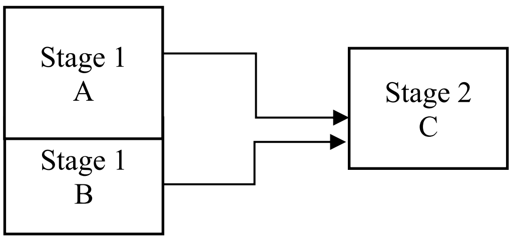

Figure 1. Two-Stage Hybrid Flow Shop Problem

The objective function in most scheduling problems is to minimize the total completion time or the maximum flow time. Despite the importance of hybrid flow shop scheduling problems as multi-objective models, studies in this area are fewer compared to other scheduling domains (Lei et al., 2022). Therefore, the most important actions envisioned for this research are:

• A two-stage HFS scheduling problem with sequence-dependent setup times in stage (1) of the hybrid job shop.

• The scheduling problem is executable where there are two identical parallel machines in the first stage and a single machine in the second stage.

• The processing time of each job in the first stage is very small and can be negligible, while sequence-dependent setup times exist between any two adjacent jobs processed in the first stage.

• When a job is completed in the first stage, it must be processed immediately on the machine in the second stage; otherwise, jobs will face delays.

• The overall modeling aims to increase lifespan by providing maintenance schedules to maximize throughput.

According to the aforementioned objectives and importance, the most significant contribution of the current research is the combination of mathematical optimization models with metaheuristic algorithms to dynamically optimize the number of human activities and other parameters of the production system in real-time. By using historical data and metaheuristic algorithms, system performance patterns can be identified, and accurate predictions regarding human resource needs and activity completion times can be provided.

The remainder of the paper is organized as follows: Section 2 provides a literature review to identify research gaps based on prior studies. Section 3 discusses the research modeling. Section 4 presents the findings from applying the proposed model. Finally, Section 5 provides a general conclusion along with research suggestions.

2) Literature Review

This section reviews the research background of activities undertaken in the discussed areas. Due to the nature and historical progression of solving the present problem, each of the relevant sections will be examined in pairs, and then, the integration of each of these topics will be presented. The category of production is highly sensitive due to its strong dependence on machinery and equipment and shows fluctuations in response to changes in machinery. This issue goes so far that the failure or improper functioning of machinery and equipment has an adverse effect on production hours, production costs, and production quality. Since the concept of integrating production and maintenance was introduced, there has always been a desire to determine a suitable mechanism for connecting and coordinating these two fields. In the following, studies that have made efforts in this area are presented. For example, Ahmadi et al. (2016) investigated the integration of production, maintenance and repair, and the quality of an imperfect process in a multi-product, multi-period system. In their model, the production system is considered an imperfect machine whose status is defined as controlled and uncontrolled. Their main goal is to minimize all costs while meeting all demands. In their modeling, reducing operating costs, preventive maintenance costs, return costs, production and setup costs, and inventory costs were considered. Their model effectively shows the interaction between quality, production, and maintenance parameters. Moreover, this model is highly efficient in finding the best balance between costs. In this research, they concluded that the higher the level of preventive maintenance and repair, the lower the quality costs. Additionally, if the costs of preventive maintenance and repair are not offset by a reduction in quality costs, the preventive maintenance and repair performed are not justifiable. Furthermore, the results obtained from solving the model confirm the strong relationship between production, preventive maintenance and repair, and quality.

Huang and Yu (2016) studied the integrated planning problem of maintenance and production via a periodic preventive replacement system with minimal repairs (preventive maintenance) against unplanned failures. Liu et al. (2017), in another study, modeled and optimized an integrated production and maintenance planning. This study continued by considering minimal repairs with periodic replacements. Using mixed-integer linear programming, they were able to minimize the optimal total cost over a limited horizon. Liu et al. (2018) presented a preventive remanufacturing planning for production equipment under operational constraints and imperfect maintenance based on a genetic algorithm-based method. This research aimed to investigate remanufacturing opportunities for production equipment to produce a product that meet deterministic and dynamic demands within a limited timeframe.

Wang et al. (2020) investigated a two-stage no-wait hybrid flow shop scheduling problem where sequence-dependent setup times are considered in the first stage. For this purpose, the NP-Hardness of the problem is first proven, then a lower bound based on the Hungarian method is proposed. Finally, a branch-and-bound approach, a tabu search, and three heuristic methods are developed for this problem. Extensive computational experiments are conducted to evaluate the performance of these methods. The computational results demonstrate the efficiency of the proposed methods.

Lee (2020) considered a two-machine flow shop with limited waiting time constraints between the two machines and sequence-dependent setup times on the second machine. These considered features arise from semiconductor manufacturing systems. The objective of this scheduling problem is to minimize the total tardiness. In this study, a mixed-integer linear programming model is presented to define the problem and used to find optimal solutions using the CPLEX mathematical programming tool. Since CPLEX required very long computation times, this problem has been identified as NP-complete. To overcome this property, a genetic algorithm is proposed to solve the problem in a short computational time. Computational experiments were conducted to evaluate the performance of the proposed algorithm, based on which the proposed genetic algorithm was better than other heuristics considered in the study.

Madenoğlu (2021) presented a type of flow shop scheduling problem with parallel machines. The proposed problem involves multi-stage and identical parallel machines at each stage, and sequence-dependent setup times and transportation times are considered. The objective function in this study is to minimize makespan. The Particle Swarm Optimization (PSO) algorithm is investigated to solve the problem and is compared with a genetic algorithm and a heuristic method. Numerical results indicate that PSO performs significantly better than the comparison set.

Rodríguez et al. (2022) proposed a new multi-agent approach that implements the maintenance and repair policy performed by technicians under the uncertainty of multiple machine failures. The presented approach is able to observe the status of each machine to coordinate decision-making in maintenance planning, which leads to the dynamic allocation of maintenance tasks to technicians, with different skills, across a set of machines. Shen et al. (2022) investigated the interaction between production scheduling and maintenance. For this purpose, a job scheduling problem in a two-stage hybrid sequential-parallel production system is optimized. To solve this problem, a two-stage maintenance strategy is developed and finally solved with a genetic algorithm (GA). Liu et al. (2022) proposed an estimation of distribution algorithm with multiple intensification strategies to solve a common type of hybrid flow shop scheduling problem. For this purpose, a two-stage heterogeneous flow shop scheduling problem was investigated.

Ullah et al. (2024) addressed cost-based hybrid flow shop scheduling optimization using a learning-based optimization algorithm. Kuang et al. (2024) used a two-stage neighborhood search algorithm to solve the hybrid flow shop scheduling problem. Yigit et al. (2025) utilized a hybrid approach for multi-criteria optimization of sequence-dependent setup-based flow shop scheduling. Zhou et al. (2024) addressed the flexible flow shop scheduling problem based on an improved grey wolf algorithm. Shahsavari-Pour et al. (2024) applied a new Pareto optimal algorithm for the flow shop scheduling problem.

Zhang et al. (2025) proposed an integrated control model of maintenance and production processes, considering the interaction between production and deterioration in a single-machine system. For this purpose, an improved value iteration algorithm was developed to reduce the computational space.

Sensitivity analysis highlights the key parameters affecting optimal policies and provides valuable insights for practical applications. Tang et al. (2025) developed an integrated optimization model for maintenance, spare parts management, and operation for a multi-component single-machine system. For this purpose, an improved mathematical-based algorithm was proposed, in which an initialization method and four local search operators were designed to improve solution efficiency.

2-1) Research Gap

Based on the above, traditional production planning models, often based on linear programming, have been well-developed in recent years (e.g., Kuang et al., 2024; Ullah et al., 2024; Yigit et al., 2024). However, the integration of these models with support activities, such as maintenance planning, is an issue that has only recently been explored (Shahsavari-Pour et al., 2024; Zhou et al., 2024). Given that proper coordination in production and maintenance can determine the good performance of a production system, the importance of integrated production and maintenance planning becomes more evident than ever. In the review of prior literature and research, most studies conducted on combining supply chain, quality control, maintenance, and production planning have been examined (e.g., Liu et al., 2018; Wang et al., 2020). Some of the reviewed research deviates from the existing reality in industrial environments and lacks the necessary efficiency for use in practical problems (Li et al., 2020; Madenglo, 2021). Therefore, the practical and effective use of these approaches becomes very difficult and challenging. With an awareness of the weaknesses and strengths of previous research, we aim to examine a study that is largely fluid and practical in terms of its structure. What is evident among the conducted studies is the lack of an optimization model for scheduling production activities using maintenance to reduce maintenance costs and increase equipment reliability and delay in manufacturing and assembly to produce a quality product within a single model. Therefore, in this research, by presenting a mathematical model and solving it through deterministic and metaheuristic methods, steps are taken to improve the efficiency of production and maintenance technology in production complexes. For this purpose, the main contribution of this research is to connect maintenance planning and production activity scheduling through the provision of a comprehensive planning. This planning, applied to a production system, ensures that maintenance on any one machine does not lead to the unavailability of all machines. A complete maintenance policy is assumed, and by solving this problem, two results are obtained: minimizing total cost and increasing system reliability. Therefore, the most important contributions are:

- Presenting a two-stage hybrid scheduling problem with sequence-dependent setup times in both stages, where first-stage activities are performed without interruption. If the single machine in the second stage is busy, it faces delays in processing jobs.
- Employing a problem-specific algorithm to obtain exact solutions for small-scale problems and developing a metaheuristic algorithm for large-scale problems to generate high-quality solutions within reasonable computational time and logical computational error. Additionally, the performance of the proposed methods is tested and validated through extensive computational experiments on randomly generated problem instances. Table 1 presents a classification for prior studies.

Table 1. Classification of Research Literature

Researcher/ Researchers	Year	Schedule	Maintenance and Repair policy			Planning horizon		Mathematical Model		Mathematical model		Solution method					Other considerations		
												Exact method		Metaheuristic method					
			Preventive	Predictive	Corrective	Short-term	Medium-term	Deterministic	Non-deterministic	Deterministic	Non-deterministic	Lagrange	Benders	NSGA-II	MOGWO	MOPSO	Reliability	Failure	Cost
Wang et al.	2020	√			√						√				√				√
Lee et al.	2020			√		√				√		√				√	√		√
Madenoğlu et al.	2021	√		√						√	√	√				√		√	
Rodríguez et al.	2022	√		√						√		√						√	√
Liu et al.	2022	√								√					√		√		
Shen et al.	2022	√			√					√					√		√	√	
Ullah et al.	2024	√									√	√						√	
This research	2025	√	√			√	√			√					√		√	√	√

3) Research Method

This research proposes a planning model to minimize the number of tardy jobs in single-machine scheduling problems with probabilistic maintenance activities. To this end, the initial assumptions for implementing the model are considered as specified below. There are M machines in the production unit, with one specific machine in each department. Each day consists of T work shifts with a specified number of hours. The number of employees or human resources is also a specified quantity equal to K. Each machine has a specific failure rate, and maintenance and repair operations must be performed on it for each operation cycle. Based on the failure rate and preventive maintenance and repairs, the machine's production rate will differ before and after maintenance and repairs. It is evident that performing these preventive repairs leads to an increase in the machines' production rate. Regarding the allocation of personnel to machines, the minimum number of employees must be considered for each cycle.

Given the nature of the problem, which falls under mathematical modeling, we will have assumptions instead of predetermined hypotheses. The assumptions for this problem are stated as follows:

- Each period has work shifts (a maximum of two shifts).
- Human resources with specific skill levels can operate the machines.
- Each part is processed by one human resource with the required skill level using one machine.
- It is not mandatory for the part to be processed in every work shift; the model determines this.

- Human resources can be active or inactive in each period (take leave, get sick, etc.).
- The more machines are used, the higher their failure rate.
- Each machine is assigned only one human resource per period; in other words, no machine has two human resources.
- The machine's failure rate is calculated based on the processing time share.
- Performing maintenance and repairs reduces this failure rate.
- Maintenance and repair time is performed within one work shift.
- A time capacity was considered for each human resource.

To describe the mathematical model of the research, the indices and sets are first introduced. Subsequently, the parameters and variables are introduced, and finally, the relationships of the mathematical model are described.

Sets:

I : Set of parts

M : Set of machines

K : Set of human resources

J : Set of shifts

T : Set of periods

U_k : Set of machines that can be processed by human resource k

U'_t : Set of human resources k available in period t

Indices:

$i \in I$: Index for the parts assembly $i = 1,2,3$

$m \in M$: Index for the set of machines $m = 1,2,3,4$

$k \in K$: Human Resources Index $k = 1,2,3,4,5$

$j \in J$: Index for the set of shifts $j = 1,2$

$t \in T$: Index for the set of courses $t = 1,2,3$

Parameters:

MM : Very large number

Cxx_{imjt} : Cost of producing part i with machine m in shift j in period t

Cx_{imkjt} : The cost of employing human resource k to produce part i with machine m in shift j in period t

cz_{mjt} : Cost of performing preventive maintenance on machine m in shift j during period t

$\tilde{d}_{it}$: Demand for the i-th part in the t-th period

cap_{tk} : Maximum available time for each human resource k in period t

t_{imk} : Processing time of part i by human resource k with machine m

η_{mt} : Percentage reduction in failure rate based on the net performed for machine m in period t

$\tilde{\lambda}^{0}_{tm}$: Initial failure rate of machine m in period t

CI_{it} : Cost of product i's inventory quantity in period t

$\tilde{s}_{imk}$: Preparation time of the i-th part by human resource k on the m-th machine

$\tilde{\gamma}_{k}$: Phase factor for the increase in the learning skill of each individual k

E_{t} : The importance of parallel system reliability in period t

Variables:

In this research, variables are classified into two types. The first category consists of continuous variables, and the second category consists of binary variables.

Continuous variables:

I_{it} : Quantity of product i in period t

xx_{imkjt} : The quantity of part i produced by human resource k with machine m in shift j during period t

y_{tmk} : Processing time by human resource k in period t by machine m

λ_{tm} : The failure rate of machine m after repeated use over period t

λ'_{tm} : Failure rate for machine m only after maintenance in period t

λ'_{tm} : Failure rate for machine m before maintenance is performed at time t

$\lambda\lambda_{tm}$: Failure rate, which includes before and after maintenance

R_{t} : Overall system reliability at time t

β'_{kt} : The learning rate of individual k after attending a class or course

β_{kt} : Learning rate of individual k in period t

B. Binary variables:

x_{imkjt} : If k human resources are used to produce part i with machine m in shift j in period t, then 1, otherwise 0.

z_{mjt} : If machine m is maintained in shift j during period t, it is 1, otherwise 0.

$z'_{mjtt'}$: If machine m performs task j in period t, and subsequent periods are indexed by nt, then 1, otherwise 0.

$zz'_{mjtt'}$: If machine m is not performed in shift j in period t, and in previous periods with index nt, then 1, otherwise 0.

zm_{kt}: If human resource k is trained in period t, then 1, otherwise 0.

Objective Functions:

In this section of mathematical modeling, we introduce the dual objectives of the research. In this research, the investigation of two objectives is simultaneously examined. The first objective, shown in Equation 1, minimizes the total cost. The total cost includes the sum of production cost, human resource utilization cost, inventory cost, and maintenance and repair implementation cost.

$$A=\sum_{t\in T}\sum_{j\in J}\sum_{m\in M}\sum_{i\in I}xx_{imjt}.Cxx_{imjt}+\sum_{t\in T}\sum_{i\in I}I_{it}CI_{it}+\sum_{t\in T}\sum_{j\in J}\sum_{m\in M}\sum_{i\in I}\sum_{k\in K}x_{imkjt}Cx_{imkjt}+\sum_{t\in T}\sum_{j\in J}\sum_{m\in M}z_{mjt}.cz_{mjt} \tag{1}$$

Additionally, the second objective of this research is to increase the impact of reliability, which will be calculable through the relationship.

Limitations:

All research limitations are introduced below.

$$I_{it}=I_{it-1}+\sum_{j\in J}\sum_{m\in M}\sum_{k\in K}xx_{imkjt}-\tilde{d}_{it} \qquad \forall i\in I,t\in T \tag{2}$$

$$\sum_{m\in U_k}x_{imkjt}\le 1 \qquad \forall i\in I,t\in T,j\in J,k\in K\cap U'_t \tag{3}$$

$$xx_{imkjt}\le x_{imkjt}.MM \qquad \forall i\in I,t\in T,j\in J,k\in K\cap U'_t \tag{4}$$

$$xx_{imkjt}\ge x_{imkjt} \qquad \forall i\in I,t\in T,j\in J,k\in K\cap U'_t \tag{5}$$

$$y_{tmk}=\sum_{i\in I}\sum_{j\in J}[\tilde{s}_{imk}+(t_{imk}).xx_{imkjt}].e^{-t.\beta_{mt}} \qquad \forall m\in U_k,t\in T,k\in K\cap U'_t \tag{6}$$

$$y_{tmk}\le cap_{tk} \qquad \forall m\in U_k,t\in T,k\in K\cap U'_t \tag{7}$$

$$\lambda_{tm}=(1+\sum_{t'\in T}\sum_{k\in K\cap U'_t}\frac{y_{t'mk}}{\sum_{i\in I}t_{imk}}).\tilde{\lambda}^0_{tm} \qquad \forall m\in M,t\in T \tag{8}$$

$$\sum_{t\in T}\sum_{j\in J}z_{mjt}\le 1 \qquad j\in J \tag{9}$$

$$xx_{imjt}\le(1-z_{mjt}).M \qquad \forall i\in I,t\in T,j\in J,k\in K\cap U'_t,m\in U_k \tag{10}$$

$$R_t=1-\prod_{m\in M}(1-e^{-t.\lambda\lambda'_{tm}}) \tag{11}$$

$$\sum_{t'\ge t}zz_{mjtt'}=z_{mjt}.(|T|-t+1) \qquad \forall m\in M,t\in T \tag{12}$$

$$\sum_{t'<t}zz'_{mjtt'}=z_{mjt}.(t-1) \qquad \forall m\in M,t\in T \tag{13}$$

$$\lambda'_{t'm}\ge\lambda_{tm}-\sum_{j\in J}zz_{mjtt'}.\eta_{mt}-MM.(1-\sum_{j\in J}zz_{mjtt'}) \qquad \forall m\in M,t,t'\in T,,\ t'\ge t \tag{14}$$

$$\lambda''_{t'm}\ge\lambda_{tm}-MM.(1-\sum_{j\in J}zz'_{mjtt'}) \qquad \forall m\in M,t,t'\in T,,\ t'<t \tag{15}$$

$$\lambda\lambda_{tm}=\lambda'_{tm}+\lambda''_{tm} \qquad \forall m\in M,t\in T \tag{16}$$

$$\beta'_{kt} = \beta_{kt} + \gamma_k . zm_{kt} \qquad \forall k \in K, t \in T \qquad (17)$$

$$\sum_{t \in T} zm_{kt} = 1 \qquad \forall k \in K \qquad (18)$$

Equation (2) determines the demand for each product and the inventory level of each product based on shifts and machines. Constraint (3) ensures that human resources are allocated based on their expertise level per shift in each period. Constraints (4) and (5) define the relationships between the two variables. Constraint (6) calculates the total processing time per machine per shift based on the skill level of the human resources. Equation (7) ensures that the maximum working time per shift is observed. In equation (8), the failure rate is calculated based on the utilization of each machine. Using Constraint (9), the time for preventive maintenance is calculated. In Constraint (10), the duration of preventive maintenance, during which there should be no production on that machine and shift, is calculated. In Constraint (11), the system reliability in a parallel system at time t is calculated. Constraints (12) and (13) specify only the times after maintenance and repair when the failure rate should decrease. Constraints (14) and (15) specify only the times before maintenance when the failure rate is the same. In Equation (16), the failure rate is calculated only before maintenance is performed. In Equation (17), the increase in an individual's skill learning coefficient is ensured based on holding training courses. In Constraint (18), the timing of training courses is determined.

3-1) Problem-Solving Method: MOGWO Metaheuristic Approach

In this paper, the MOGWO metaheuristic algorithm is used to solve the model. This algorithm is the multi-objective form of the Grey Wolf Optimizer algorithm. The Grey Wolf Optimizer algorithm was presented by Mirjalili et al. (2014) at Shahid Beheshti University. This algorithm, such as PSO and ACO algorithms, is a swarm intelligence algorithm and uses only one (combined) operator to determine the position of wolves in the problem-solving space.

Grey wolves are creatures that live in a semi-democratic manner, and the position of each wolf in their society is known. In this society, the alpha wolf, its successors, and other wolves are recognized. Grey wolves live and hunt in packs. Each pack of grey wolves typically consists of 7 to 12 wolves. These packs are among the most dangerous predators. To hunt, grey wolves first encircle and surround the prey, and then, begin to tire the prey by tightening the encirclement. Then, they take turns attacking the prey under the command of the alpha wolf, and finally bring the prey down. The figure below shows an example of wolves attacking prey in the real world. As mentioned, grey wolves live in a strict hierarchy; therefore, the figure below shows the hierarchical structure of grey wolves.

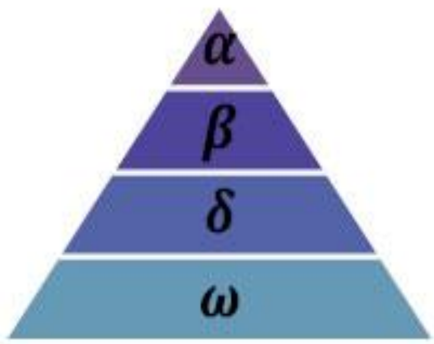

Figure 1. Hierarchical Structure of Grey Wolves

- The Alpha pair, known as the leaders of the pack, are responsible for making decisions regarding hunting, sleeping locations, waking times, and so on. The Alpha's decisions are enforced on the entire pack. However, a type of democratic behavior is also observed.

- The second rank in a pack's hierarchy belonging to the Beta wolves. Beta wolves assist the Alpha in decision-making and other pack activities. These wolves are the best candidates to become Alpha when the current Alpha becomes very old or dies.

- The wolves with the lowest rank are the Omega wolves. This group of wolves plays the role of a sacrificial lamb in the pack. They must be submissive to all other wolves and are also the last wolves to eat. It seems that Omegas have low importance in the pack, but it has sometimes been observed that if Omegas are lost, the entire pack experiences problems and internal conflicts.

Wolves not mentioned in the hierarchy above are called Delta wolves. Delta wolves are under the command of the Alpha and Beta but are superior to the Omega. Based on these points, the GWO algorithm was formulated. The flowchart of the GWO algorithm is as follows:

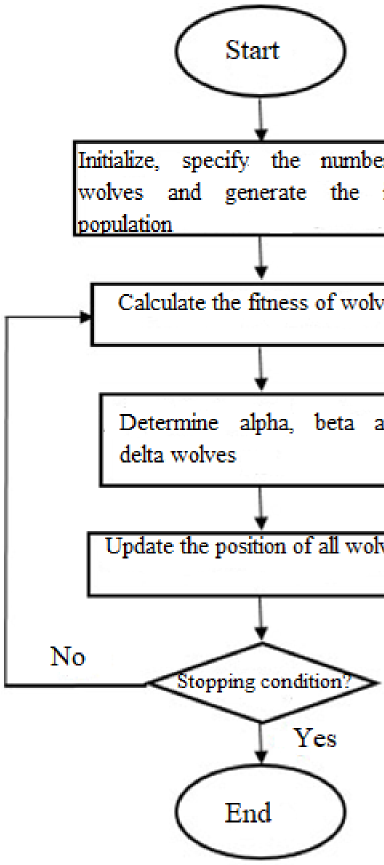

Figure 3. Grey Wolf Optimization Algorithm Flowchart

4) Findings

4-1) Case Study: Mazandaran Steel Company

Mazandaran Steel Factory was established in 1983 with the aim of producing steel sheets to support the construction and automotive industries in the north of the country, located on the Goharbaran road near Dasht-e Naz Airport in Sari. This factory is part of the basic industries sector and is considered the largest steelmaking unit in the north of the country. Throughout its years of operation, Mazandaran Steel Factory has managed to gain an excellent position in the country's steel market due to the production of high-quality and marketable products, becoming one of the largest producers of steel sheets in Iran. In this research, through examination, study, and interviews with the managers of Mazandaran Steel Company, and by presenting various production problems and issues, as well as coordinating with other influential supporting departments where a lack of coordination, rework, parallel work, and actions for improvement and integration have sometimes occurred, the model assumptions were determined and explained, considering the company's overall operation. Mazandaran Steel Company produces various products such as ingots, billets, slabs, and blooms. These productions are carried out using expensive equipment with high cost and production sensitivities, and production must be continuous and ongoing in each period. The failure of parts of this equipment due to various reasons, such as friction and high wear and tear, is significant. The manufacturing workshop of this company consists of various machine tools, such as different types of lathes, grinding machines, various milling machines, and drills. The manufacturing of these parts is carried out in different periods and shifts by human resources with varying levels of technical skills. Parts such as poles, shafts, and gears are vital and strategic components of the steel manufacturing company that must be continuously and constantly produced. Field surveys and interviews with managers, supervisors, and experts were conducted to obtain precise information for model development.

4-2) Practical Numerical Results

The results obtained from solving the model are categorized into two dimensions: production planning and human resource scheduling in a job shop problem. In this section, we discuss the results obtained in these two dimensions.

a) Production Planning

The problem, created in GAMS software, using the CPLEX tool, was able to find the optimal solution for the problem. Additionally, by running the mathematical model, the optimal values for the decision variables were obtained. Tables 2, 3, and 4 indicate, respectively, the optimal values of inventory for product i in period t; the quantity of part i produced by human resource k with machine m in shift j in period t; and also, the processing time by human resource k in period t using machine m.

Table 2. Optimal Inventory Value of Product i in Period t

Part demand	First Period	Second Period	Third Period
i_1	45	35	40
i_2	50	50	42
i_3	30	30	30

Table 3. Optimal Production Quantity of Part i by Human Labor k with Machine m in Shift j in Period t

Piece / Human resources	First Period				Second Period				Third Period			
	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4
k_1	40	20	20	25	15	10	0	6	34	0	35	11
k_2	40	15	25	25	20	10	0	6	30	0	35	11
k_3	35	15	30	30	15	10	0	6	30	0	35	11
k_4	55	20	35	30	20	14	0	14	37	0	42	17
k_5	56	20	35	35	25	14	0	14	37	0	40	15

Table 4. Optimal Processing Time for Human k in Period t by Machine m

Piece / Human resources	Piece 1				Piece 2				Piece 3			
	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4	m_1	m_2	m_3	m_4
k_1	10	2	2	3	١٠	3	0	3	20	0	6	2
k_2	10	2	2	3	10	3	0	3	20	0	6	2
k_3	10	2	2	3	10	3	0	3	20	0	6	2
k_4	15	2	2	2	15	6	0	5	18	0	4	3

$\boldsymbol{k_5}$	15	2	2	2	15	6	0	5	18	0	4	3

In Table 5, the optimal failure rate for the conditions after repeated use, before and after maintenance and repairs under the planned program, is calculated for machinery m in period t.

Table 5. Failure Rate Status

Breakdown	Machinery	First period	Second period	Third period
After repeated use	m_1	55	4.5	4.5
	m_2	4	4	3.5
	m_3	3	3	3
	m_4	2.5	3	3
Before maintenance and repairs	m_1	4.25	3.25	3.65
	m_2	3.75	3.75	1.75
	m_3	2.5	3	3
	m_4	2.12	3	3
After maintenance and repairs	m_1	4	2	2
	m_2	2.1	1.1	1
	m_3	1.1	1	1
	m_4	1.5	1.5	1.5

According to Table 4, it is observed that the failure rate after repeated use of machinery in each period is significantly higher than when performing maintenance and repairs and not performing them. Furthermore, Table 4 shows that if maintenance and repairs are carried out, the failure rate is lower compared to before performing maintenance and repairs. Additionally, Table 6 shows the learning rate of individual k in period t, as well as the learning rate of individual k after attending a class or course.

Table 6. Learning Rate Status

Human resource	Individual learning rate	Learning rate after training class
$\boldsymbol{k_1}$	0.50	0.65
$\boldsymbol{k_2}$	0.48	0.59
$\boldsymbol{k_3}$	0.40	0.62
$\boldsymbol{k_4}$	0.60	0.78
$\boldsymbol{k_5}$	0.55	0.78

Based on Table 5, it is clear that after conducting training classes among the influential human resources in the production process, the rate of specialized training learning for human resources increases.

b) Human Resource Scheduling

By implementing the model in GAMS software and verifying the feasibility of the problem, an optimal solution to the problem can be obtained. This model has been solved for seven days a week, based on two work shifts, considering five workers. For a better understanding of the output, the results of the model solution are presented in Tables 6, 7, and 8. To design these tables, workers were first numbered from 1 to 5, and then, these tables specified the numbers of workers who should be working on the respective machine in each day and each shift. For example, Table 6 specifies that on machine 1, in the first shift on day 1, workers 3, 5, and 2 should work. Or in Table 7, on machine 2, in the first shift on day 1, workers 1 and 2 should work. Additionally, in Table 8, on machine 3, in the first shift on day 1, workers 2 and 5 work. Similarly, the assignment of workers for each day of the week in each work shift on each machine is determined in Tables 7, 8, and 9. Given the obtained assignment, it is clear that if a worker is used in any shift on any day, that worker will not be used in the next shift. This can be justified by using the zero-one programming incorporated into the proposed model.

Table 7. Scheduling Plan for Machine 1 in Department 1 in Each Day and Each Shift of the Planning Horizon

Days/Shifts	Shift 1	Shift 2
Day 1	3, 4, and 2	1 and 5
Day 2	1, 4, and 5	3 and 2
Day 3	3, 4, and 5	5 and 2
Day 4	3, 4, and 1	5 and 4
Day 5	3, 2, and 5	1 and 4
Day 6	3, 1, and 5	4 and 2
Day 7	1, 2, and 3	4 and 5

Table 8. Scheduling for Machine 2 in Department 2 in Each Day and Each Shift of the Planning Horizon

Days/Shifts	Shift 1	Shift 2
Day 1	3, 4, and 5	1 and 2
Day 2	2, 4, and 5	1 and 3
Day 3	1, 3, and 2	4 and 5
Day 4	2, 5, and 4	1 and 3
Day 5	1, 4, and 3	5 and 2
Day 6	1, 5, and 2	3 and 4
Day 7	1, 3, and 4	5 and 2

Table 9. Scheduling Plan for Machine 3 in Department 3 in Each Day and Each Shift of the Planning Horizon

Days/Shifts	Shift 1	Shift 2
Day 1	3, 4, and 1	2 and 5
Day 2	2, 5, and 1	3 and 4
Day 3	3, 4, and 2	1 and 5
Day 4	3, 4, and 1	2 and 5
Day 5	2, 4, and 5	1 and 3
Day 6	3, 1, and 5	2 and 4
Day 7	1, 2, and 5	3 and 4

4-3) Comparison of Exact and Metaheuristic Results

This section compares the results of the proposed model with the Grey Wolf metaheuristic algorithm. The algorithm's performance is determined based on two concepts: first, quality, and second, response speed. Additionally, the difference in the objective function value of the Grey Wolf algorithm and the objective function value presented in GAMS as well as the solution time speed of these two methods are compared. To evaluate the Grey Wolf algorithm used for this problem, coding was done in MATLAB software. Subsequently, 7 problems of different sizes were generated based on changes in the number of human resources.

In Table 9, the results obtained from the exact solution of the generated examples using GAMS software are compared with the results obtained from the Grey Wolf algorithm based on two criteria: time and objective function value. Since the solution time of GAMS software for high-dimensional problems is very long, a time limit of 3600 seconds or 1 hour has been considered for it. It should be noted that if solving the problem in GAMS software requires more than 1 hour, GAMS software will provide a feasible (not necessarily optimal) solution upon reaching 1 hour, and the program execution will terminate. Table 10 summarizes the comparative results of GAMS software with the multi-objective

Grey Wolf algorithm based on experimental execution in small, medium, and large dimensions, compared with the number of parts, machines, human resources, shifts, and period.

Table 10. Results of Solving Sample Problems with GAMS and the Grey Wolf Algorithm

Issue size	Issue Number	Number of Human Resources	Number of pieces	Number of machines	Number of periods	Number of shifts	Exact solution with GAMS software		Multi-hyphenate grey wolf algorithm		GAP (%)
							Objective function	Solution time (seconds)	Objective function	Solution time (seconds)	
Small size	PR1	1	1	1	1	1	6880	125	6880	57.7	0%
	PR2	2	1	1	1	1	7589	140	7589	65.32	0%
Medium size	PR3	3	2	2	2	1	3520	1330	3520	179.827	0%
	PR4	4	2	2	2	1	3965	2850	2932	263.8	2%
	PR5	5	2	3	2	2	4187	3360	4062	394.9	0.02%
Large size	PR6	6	3	4	3	2	-	-	5564	567.4	-
	PR7	7	3	4	3	2	-	-	6037	664.7	-

According to the results obtained from the computations, the mathematical model shows very good capability in solving small-scale problems. This holds true even when the number of human resources is assumed to be three, despite the increase in computational time and, consequently, the problem dimension. This is because the gap between the results of the exact solution and the metaheuristic model is reported as zero percent. However, with a further increase in the number of human resources and an increase in problem complexity, while computational time increases, the gap between the results also increases to an acceptable extent. To the point where, when the human resources are considered to be six to seven, solving the problem becomes so difficult for the proposed model that it is unable to compute and solve it. This point also applies to increasing the number of parts, the number of machines, the number of periods, and the number of shifts. However, given that in small and medium problems, the gap between the mathematical model and the metaheuristic model was negligible, we can rely on the results provided by the metaheuristic model for solving large-scale problems, trusting the computed results.

4-4) Sensitivity Analysis

In this section, the numerical results are used to examine the impact of two important parameters: the coefficient of increase in machinery failure before maintenance and repair, and the coefficient of decrease in failure after maintenance and repair. For this purpose, the values of each of these parameters were varied between -20% and +20%, and the objective function value was reported accordingly. Figures 4 and 5 graphically illustrate the impact of these two parameters in the mathematical model. According to Figure 4, if the machinery failure rate before maintenance and repair decreases by 20%, the objective function value reaches its maximum possible value. Additionally, if it increases by 20%, the objective function value reaches its minimum possible value. Furthermore, in Figure 5, if the failure rate coefficient after maintenance and repair decreases by 20%, the objective function value reaches its minimum possible value, and if it increases by 20%, the objective function value reaches its maximum possible value.

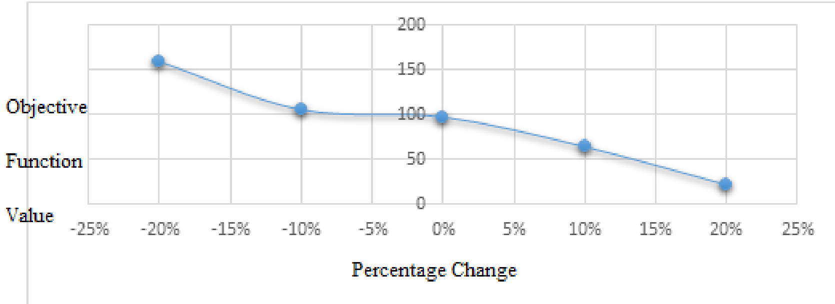

Figure 4. The Effect of the Failure Rate Increase Coefficient on the Objective Function

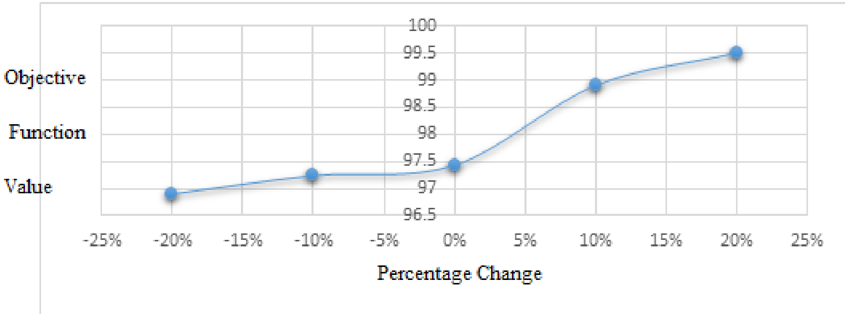

Figure 5. The Effect of the Damage Reduction Factor on the Objective Function

5) Conclusion

This research aimed to minimize tardy jobs in single-machine scheduling problems to improve the performance of data processing systems and reduce machine activity execution time. In this study, mathematical programming methods for solving the problem of minimizing tardy jobs in single-machine scheduling were investigated, which were solved using a deterministic method and epsilon-constraint. By employing the proposed framework, we have provided a suitable schedule for deploying appropriate human resources in each work shift for maintenance and repair programs, as well as an optimal production plan throughout the process by determining the optimal production quantity. Furthermore, by adhering to the provided schedule, we will be able to determine the maximum reliability for machinery. The inventory level in different periods, the optimal quantity of parts produced by human resources with machines in different time periods, the optimal time for process execution in each time period for manufacturing each part, and finally, the status of the failure rate of each device in each time period for various maintenance and repair activities have been calculated. Additionally, the status of the learning rate after training periods and its impact on the device's failure rate have been examined.

For studies like the current research, focusing on minimizing tardy jobs in single-machine scheduling problems along with probabilistic maintenance activities at Mazandaran Steel company, it is suggested to use optimization approaches such as the framework proposed in this research. This is because optimization tools like deterministic and even metaheuristic algorithms with constrained search can help managers improve the performance of planning systems. Additionally, by using these optimization tools, managers can find the best solution to minimize tardiness in single-machine scheduling tasks and probabilistic maintenance activities. To this end, they can test their system model using historical and newer data related to system parameters, and then, use optimization methods to make the best decisions to reduce tardiness and optimize probabilistic maintenance activities. Moreover, by reviewing previous research in this field, they can examine the best optimization methods and algorithms and utilize them to improve their research.

The current research faces various limitations. For example, obtaining accurate and complete data regarding the number of activities, task execution times, and human resource capacity may be difficult. Incomplete data can lead to inaccurate results. Mathematical models may become highly complex due to the intricacies of production system and the interactions between activities and human resources. This complexity can lead to difficulties in analyzing and solving the model. Furthermore, the use of specific software for solving mathematical models may encounter limitations such as the inability to process large amount of data or incompatibility with existing systems. For further research, it is suggested to use data mining and past data analysis to gain a more comprehensive view of system performance, incorporate uncertainty into the model, estimate the model based on robust planning decisions, use metaheuristic optimization methods, and compare them with the results of the deterministic method.

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