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Increasing the Speed of Processing Supply Chain Decisions in Uncertain Conditions Using the Internet of Things

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ABSTRACT

New technologies have enabled companies to respond to market needs quickly and efficiently. In this context, the new concept of virtual network chain was formed. This research has studied a multi-period and multi-product network to the Internet of Things using virtual empowerment with the aim of maximizing profit and minimizing the processing speed of uncertain data. In addition to the usual cost, the virtualization cost is considered in the objective. To achieve on-line decision-making design in the network, an acceptable value for the transfer time is set. Moreover, to be close to reality, the product return rate and uncertain transportation costs are considered. The virtual closed-loop network (VCLSC) is modeled with the probability of no interruption of intervals. The set has the loops of supplier, manufacturer, distributor, customer, recycling center, and organ center, and the returned products are recovered or destroyed based on the quality. The model is solved robustly and its behavior is examined by analyzing the input programs. The results indicate that the processing is highly sensitive to the change of options.

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1) Introduction

In the contemporary business environment, effective supply chain management (SCM) is paramount for organizational competitiveness. Classical SCM involves strategic, operational, and tactical decisions, and the alignment of these decisions is a critical issue for achieving optimal overall performance (Dalal et al., 2024). A key challenge is that improvement in one component of a supply chain does not guarantee the optimized performance of the entire system (Dutta et al., 2020). To address this, modern frameworks focus on continuous improvement by integrating strategic human intelligence with operational machine intelligence, ensuring decisions are adaptive to internal and external changes (Rezaei et al., 2023).

A foundational tool for this integration is the Supply Chain Operations Reference (SCOR) model, released by the Supply Chain Council in 1996, which provides a comprehensive and reliable framework for performance measurement and decision alignment (Real-time SCPM). By employing SCOR-based metrics within multi-objective optimization models, organizations can move beyond seeking isolated, low-cost solutions. Instead, they can pursue a balanced optimization of conflicting performance indicators, such as cost, reliability, and responsiveness, to create mutual cooperation among members and deliver the highest overall value (Real-time SCPM). This approach ensures that operational activities are directly connected to strategic goals, facilitating a process of continuous and simultaneous performance improvement across the supply chain (Rezaei et al., 2022).

Today, new information and communication technologies are the most important enablers for optimizing business processes and achieving integration with supply chain partners across the global landscape (Dutta et al., 2020). With these technologies, all flows of products, information, and finances are managed appropriately. One of the most important digital technologies is the Internet of Things. This technology provides a platform for financial savings in transportation and optimization of information processing in virtual supply chains. The Internet of Things allows multiple technologies with different capabilities, such as sensing, storage, connectivity, computation, monitoring, and management, to be integrated (Prasanth & Jayachitra, 2020).

The Internet of Things enhances the reliability of the supply chain by enabling event detection and data exchange in an online format and facilitates business processes. By detecting resource changes in real time, it improves the management of supply chain assets and, finally, increases supply chain agility by accelerating the flow of information (Dweekat & Park, 2016).

The virtual supply chain connects business partners on a digital platform and enables the exchange of electronic data, including sales, purchases, product movements, services, and money (Pourhamidi & Mohajerani, 2007). Timely sharing of information leads to reduced waste and increased efficiency throughout the chain. Accurate information flow equips supply chain management with the ability to understand, forecast, and respond promptly to changes in market conditions, as well as accelerate information transfer among members, which is essential for improving control capabilities, flexibility, performance, and detecting abnormal events (Liu & Sun, 2011).

Today, remanufacturing has become common in many organizations due to economic and environmental benefits, and it has even become mandatory in many countries. Reverse logistics and recycling enable companies to use their resources optimally. One of the critical factors that affects the performance of reverse logistics is consumers' willingness to return used products (Shaharudin et al., 2015). However, there are few individuals with appropriate social responsibility who return their used products to collection centers. This causes collection centers to have low performance, while the collection process is a prerequisite for an efficient closed-loop supply chain. In this situation, incentive programs are needed to encourage customers to return high-quality used products (Geyer et al., 2010).

The main objective of this article is to design a comprehensive framework that integrates the flows of materials, information, and finances in the closed-loop virtual supply chain in an online manner by increasing the speed of information processing. In addition, for the simultaneous optimization of profit along with increasing the speed of data processing, virtualization costs, such as IoT energy consumption, product recall, virtual supply chain information security, along with other supply chain costs, are considered in the model. Our main approach in increasing speed has been to add the maximum allowable delay time in decision processing.

2) Literature Review

The virtual supply chain is an organizational structure that facilitates the efficient and effective flow of physical goods and information in an integrated manner, and it is distinguished from the traditional supply chain due to its flexibility in quickly adopting and adapting to changes in the business environment (da Cruz Caria, 2016). The traditional tracking system largely relies on paper-based or internal computer systems. Paper-based registration is time-consuming and prone to errors (Rezaei & Babazadeh, 2020). Virtualization provides the possibility of tracking and monitoring products and their history. Companies achieve operational efficiency by virtualizing their processes (Yadav & Misra, 2019). One of the keys to a successful virtual supply chain is the timely and accurate exchange of information with software programs; given that the virtual supply chain relies on an effective communication system, it requires the development of an appropriate information system using various information technologies (Gunasekaran & Ngai, 2007).

Hongze and Davidrajuh (2005) have investigated the use of an iterative method for designing the distribution chain in an agile virtual environment. They examined two strategic models and one tactical model. The strategic model specifies the location of distributors, and then, based on the output of the strategic model, the tactical model determines inventory planning and vehicle routing between different nodes of the chain. Pishvaee et al. (2011) present a multi-objective mixed integer linear programming model that includes maximizing network responsiveness and minimizing total costs in the closed-loop supply chain network and have used a genetic algorithm to address the designed model.

Listeş and Dekker (2005) have proposed a scenario-based stochastic programming model for designing an integrated direct/reverse supply chain network, and a decomposition method for solving the model in large-sized instances based on the branch and cut procedure has been presented. Min and Ko (2008) have developed a multi-period mixed integer linear programming model for designing a multi-product closed-loop logistics network with third-party logistics1 participation. This model is used to determine the number and location of facilities for repairing returned products from retailers or end customers, redistribution, inspection, repair, and renovation. To solve the model, a genetic algorithm has been developed.

Du and Evans (2008) presented an advanced bi-objective mixed-integer linear programming model that integrates distribution centers with collection centers and recovery centers for designing a closed-loop logistics network of third-party logistics services. They proposed a hybrid scatter search method to solve the presented model. Another article that well addresses the integrated design of forward and reverse logistics networks is conducted by Lee and Dong (2008). In this paper, a type of hybrid facility is used that plays both the role of distribution centers (warehouses) in the forward flow and the role of collection centers in the reverse flow for designing the logistics network of computer products. This problem is modeled using mixed-integer linear programming and, due to its high complexity, is solved using a heuristic method combined with the metaheuristic tabu search method.

Additionally, Pishvaee et al. (2011) presented a mixed-integer programming model for designing a closed-loop supply chain network that can support recycling and disposal activities. The network considered in this paper includes customers in two first- and second-tier categories, collection/inspection centers, refurbishment, redistribution, and disposal with limited capacities. Amin and G. Zhang (2013) examined a general closed-loop supply chain network that includes production centers, disassembly, remanufacturing, and disposal sites. This model was the first attempt to simultaneously consider supplier selection, order allocation, and closed-loop supply chain network configuration issues. Qiang et al. (2013) also investigated a closed-loop supply chain network with centralized decision-makers including raw material suppliers, retail markets, and manufacturers who directly collect recycled products from the demand market.

1. 3PL

Pishvaee and Torabi (2010) addressed the integration of network design decisions in both forward and reverse supply chain networks in their proposed model, while also combining strategic network design decisions with tactical material flow decisions. The objectives considered include minimizing both total costs and total delays in product delivery. This research is distinguished from other articles by introducing a comprehensive model that supports recovery and refurbishment processes.

Verdouw et al. (2013) investigated how to apply the Internet of Things concept to enhance supply chain virtualization in the floristry sector. They developed a conceptual framework for analyzing supply chain virtualization and applied it to the Dutch flower and plant sector to examine the current state and identify future virtualization challenges in the flower industry. Li et al. (2014) examined the quality of IoT services in a multi-objective programming model, considering web service quality attributes (execution time, reliability, and execution cost) as web service quality evaluation criteria, with regard to IoT domain applications.

Helo et al. (2014) implemented a cloud1-based supply chain virtualization prototype and presented a practical solution for initiating new orders, resource selection, planning, activity control, and parts production in the provided model. Long (2014) investigated supply chain virtualization networks through a combination of computational testing and operational technology. Moreover, from a methodological perspective, rather than an applied one, he proposed a distributed agent-based computational testing framework based on material, information, and time aspects, along with implementation solutions for expanding supply chain virtualization networks. Zaballos et al. (2014) introduced a stochastic model for designing a closed-loop supply chain network under demand and raw material supply uncertainty, which integrates network design decisions with transportation decisions such as mode of transport selection.

Fang et al. (2015) presented a three-stage integrated model based on IoT technology to optimize raw material procurement, production, product recycling, pricing, and profit strategy for product supply at each stage of the life cycle. Ramazani et al. (2014) provided a multi-product multi-period closed-loop supply chain network design model including decisions such as facility location, supplier selection, product flow allocation, and transportation mode selection. Saffar and Razmi (2015) proposed a bi-objective model considering the environmental impact of facilities for a forward and reverse supply chain, which includes decision-making regarding the location of collection, inspection, and recycling facilities, determining products in the flow at facilities, the number of machines at each facility, and the product type.

Verdouw et al. (2016) examined the role of virtualization in the food supply chain management framework from the perspective of the Internet of Things and proposed a theoretical information system architecture for implementing this scheme. Venckauskas et al. (2016) introduced a framework for modeling security, energy, and environmental issues as key features in determining the quality of services for IoT-based applications.

Yan (2017) investigated the revenue increase optimization of perishable goods supply chains using the IoT. He considered two revenue models to calculate the revenue of perishable goods supply chains before and after the application of IoT to examine its impact on this supply chain. Kakhki et al. (2018) developed a supply chain virtualization system to integrate business processes and examined the first layer of the proposed architecture scheme without any further details on applications, processes, and data flows.

Yadav and Misra (2019) examined the functioning of blockchain when integrated with virtual supply chains and its impact on various operational efficiencies, including cost reduction in food supply chains. Matsuda et al. (2020) investigated supply chain virtualization by building a cyber-physical system for a smart supply chain. They developed an organizational model that supports a data model. They used a mathematical model to describe the behavior of the integrated data model.

1. Cloud Supply Chain

Kulinska and Kulinska (2019) presented the most important changes resulting from the enforcement of the General Data Protection Regulation (GDPR) for creating and managing virtual supply chains, along with introducing risk sources related to strengthening personal data protection. Smith and Dhillon (2019) examined the application of blockchain technology to facilitate trust between various supply chain agents, emphasizing key issues of credibility, traceability, and transparency in virtual supply chain risk management. Nishi et al. (2020) proposed a general configuration method for a multi-agent virtual supply chain system using organizational electronic catalogs. They utilized a collaborative virtual supply chain configuration method.

Jennifer (2020) investigated the development of a new information processing system on the IoT platform through healthcare monitoring. She analyzed the effective use of big data in the IoT environment through the proposed architecture to achieve minimum real-time latency. He et al. (2020) studied the theoretical and practical challenges and opportunities arising from the IoT in supply chains. They examined the performance of IoT and its implications for big data analysis on supply chain performance, particularly with regard to dynamics, coordination, and optimization, using big data obtained from smart connected products.

Sallam et al. (2023), in their study, examined the challenges, opportunities, and best practices for applying the IoT in supply chain management. They highlighted challenges such as security issues, system integration, and initial costs, and identified opportunities for improving efficiency, real-time tracking, and strategic decision-making. Best practices include standardizing protocols, workforce training, and focusing on data security, which can contribute to the successful implementation of IoT in supply chains.

Vlachos and Graham (2025) in their research, through a systematic literature review and bibliometric analysis of 572 articles, explored the role of the IoT in supply chain management from the past to the future. They proposed the TCM-AIO-E framework, which covers antecedents, implementation mechanisms, and outcomes, focusing on aspects such as decision-making, visibility, traceability, and agility. The findings indicate the evolution of IoT from an efficiency tool toward a strategic one for creating autonomous and self-learning supply chains, with suggestions for future research on integration with technologies such as generative artificial intelligence.

Most previous related studies have addressed the topics of the IoT, blockchain, and supply chain optimization separately; however, the use of bi-objective or multi-objective mathematical models with an IoT approach to improve decision processing speed under uncertainty remains limited. Most conducted studies have been descriptive and have stated the important features of virtual supply chains.

Therefore, there is a need to develop models that simultaneously consider uncertainty, leverage IoT technologies, and focus on increasing information processing speed in supply chain decision making.

Given the key role of supply chain decision speed, this research examines the role of the IoT in optimizing profit and information processing speed in virtual supply chains, with emphasis on parameters of demand, return rate from the market, and transportation costs, which have significant uncertainty in closed-loop supply chains. The problem is modeled using robust optimization and solved with GAMS software. Finally, a numerical example and sensitivity analysis on the main model parameters are presented to illustrate the importance and applicability of the developed model. Table 1 provides a summary of the most important related research, and the characteristics of the current research are presented in the last row.

3) Problem Definition

The proposed virtual closed-loop supply chain network is a multi-period and multi-product network with a product tracking approach via the IoT. The components of the forward supply chain consist of suppliers, manufacturers, distributors, and customers. In the forward chain, the material flow is from the supplier to the customer, and if a product is damaged en route, it is sent to the manufacturing center for repair and ultimately stored for shipment in the next period. In the reverse supply chain, collection, recycling, and disposal centers for products are active. Returned products from customers are collected, tested, and inspected, and reusable products are sent to recycling centers. There, through disassembly and separation of reusable raw materials, they are sent to the manufacturing center, while the remaining products that are not reusable are sent to disposal centers. Additionally, products sent to recycling centers are forwarded to manufacturing or disposal centers after processing.

A proposed purchase cost for returned goods is utilized as an incentive policy to increase customers' willingness to return used products. The proposed model optimizes both profit and data processing delay in the virtual closed-loop supply chain. To this end, usual supply chain costs along with virtualization-related costs, such as security, energy consumption, invocation, and IoT facilities are considered. Information on reusable returned products is stored in an IoT database. Company information regarding the product life cycle is collected, processed, and shared via the IoT.

The manufacturer controls the entire lifespan and product life cycle data in the CLSC network through IoT-connected products. With an increase in production volume, the data processing time in the IoT system (information processing delay) also increases, indicating slower decision processing speed. Any changes in product status can be monitored by the IoT and stored in the database. This information can be obtained using various types of electronic barcode readers through a Radio Frequency Identification (RFID) tag on each product, and any IoT user can access this information as needed. Therefore, the quantity of each returned product can be evaluated and utilized (Paksoy et al., 2016).

It should be noted that the more centers are equipped with IoT facilities, the greater the volume of data generated, requiring more processing time for storage and access, which inevitably leads to greater delays in the network. Therefore, network delay time (for data processing) is also considered in the model. In this regard, decisions are made regarding the production quantity of each product, selection of appropriate suppliers, quantity of each recycled part from each recycled product, determination of the purchase price for each returned product, and the maximum allowable network delay time in each period which practically results in increased information processing speed and approximation to online decision making.

Problem Assumptions

1. The shelf life of each product is assumed to span multiple periods, after which customers can deliver the products to collection centers.
2. The price of produced products does not differ, whether they are made from primary materials, secondary materials, or a combination thereof.
3. The parameters under uncertainty include the quantity of returns, product demand, and transportation costs.
4. The inspection cost per unit of goods at the collection center affects the product's total cost.
5. In case of shortage, the manufacturer must bear the cost.
6. Holding costs for returned products at the collection center are not considered.
7. The relationship between the purchase price of returned products and the scale factor (λ) is an exponential function with parameter θ [1] ($\lambda = e^{-\frac{pr_{bpt}}{\theta}}$).

4) Theoretical Foundations and Managerial Implications

1-4 Supply Chain

A supply chain can be defined as a communication network between various sectors, from suppliers to manufacturers, from manufacturers to distributors, from distributors to customers, through production and services, in such a way that it manages the flow of materials, goods, money, and information to identify an organization's needs. There is an assumption that a supply chain should act on behalf of organizations at the forefront of competition with competitors (Ren, 2019). Therefore, it is considered a

1. This parameter represents the average customer reward from the manufacturer. It is calculated and displayed by the IoT system using the recorded data of the returned product.

strategic factor for achieving organizational goals such as competition, improving customer service, and increasing profitability (L. Zhang, 2021).

Supply chains coordinate activities in such a way that customers can obtain products and services with quality, reliability, and at the lowest cost. Facilities in the supply chain include factories, warehouses, distribution centers, service centers, and retailers. Supply chain management seeks to integrate activities and information flows by improving and coordinating procurement, production, and product delivery activities.

Mizuno (2022) states, regarding the supply chain, that it is an integrated philosophy for managing flows along the distribution channel, from the supplier to the final customer. Every business organization is part of a supply chain, and many organizations are part of multiple supply chains (Shaw, 2021). The short-term goals of supply chain management include increasing productivity, reducing inventory, and cycle time, while its long-term goal is to increase customer satisfaction, market share, and profit for all affiliated organizations in the supply chain (Chung, 2022).

Some researchers have limited the supply chain to relationships between buyers and sellers; such a perspective only focuses on first-tier purchasing operations in an organization. Another group takes a broader view of the supply chain and considers it to include first-tier and second-tier suppliers, and so on. Such a perspective on the supply chain only analyzes the supply network. In the third perspective, the supply chain includes all activities required to deliver a product or service to the final customer. With the mentioned perspective, manufacturing and distribution activities are added to the chain as part of the flow of goods and services. With this perspective, the supply chain encompasses three areas: procurement, production, and distribution (Senvar, 2019).

2-4 Closed-Loop Supply Chain

The supply chain consists of a network of suppliers, manufacturers, warehouses, distribution centers, retailers, and customers. From the customers, money and information flow to the previous components of the chain. In reverse logistics, products are also returned from customers to manufacturers. Reverse logistics refers to the process of planning, implementing, and controlling the reverse flow of raw materials, in-process inventory, packaging, and finished goods from a manufacturer, distributor, or point of consumption to a point of recycling or disposal. If we consider both direct and reverse supply chains simultaneously, the resulting network forms the closed-loop supply chain. The concept of closed-loop supply chain has attracted a great deal of attention today. The configuration of both reverse and direct supply chain networks has a significant impact on the performance of each.

3-4 Virtual Supply Chain

Today, supply chains operate in a competitive and dynamic environment that faces numerous challenges such as uncertainty, demand fluctuations, complexity, and costs (Mohammadi et al., 2022). These challenges require quick and flexible responses so that organizations can maintain their competitive advantage and meet customer expectations (Abdoli & Hadi Mokhtari, 2024). In this regard, information technology plays a fundamental role in supply chain management and helps improve communications, data collection, acquisition, and transfer for effective decision-making and improving supply chain performance.

The virtual supply chain is collaboration in a supply chain through the internet, by a dynamic network of collaborating organizations whose normal activities are based on the internet. The goal of these organizations is to exploit business opportunities to provide unique, timely goods (Scott & Mula, 2009). Today's business conditions, including greater product diversity, shorter product life cycles, and unpredictable demand levels, impose additional pressure on manufacturing companies worldwide. To cope with such a situation, geographically distributed companies use a virtual platform to work with partners (Shamsuzzoha & Helo, 2017). The virtual supply chain includes tools for managing the flow of information related to planning, sourcing, manufacturing, and delivery activities, which is supported by collecting, processing, and sharing information (Helo et al., 2016). The integration of information systems and internet technology has led to the creation of a virtual supply chain and results in improved decisions through information sharing at the decision-making level (Scott & Mula, 2009). Information sharing, collective collaboration, and long-term coordination lead to the improvement of companies' competitive advantages (Lotfi et al., 2013).

4-4 IoT and Other Emerging Technologies

The IoT is one of the newest developments and new revolutions in information technology that provides a paradigm shift in several fields, including supply chain management (Ben-Daya et al., 2017). The term Internet of Things was coined by Kevin Ashton in 1999. The IoT is a collection of physical and virtual objects that are connected to each other through a network for communication and sensing or interacting with the internal and external environment, and are digitally connected for sensing, monitoring, and interacting within a company and among other companies (Abdel-Basset et al., 2018). The IoT refers to the possibility of all objects communicating with each other and with humans, along with their identification and discovery in an integrated network with a specific identifier, and provides the possibility of connecting anyone to anything at any time and place (Hashemi & Sotoudeh, 2020).
The IoT technology has completely changed the environment in which supply chains operate. Vast amounts of data and information spread faster in real time throughout the supply chain, and the efficiency of discovering and utilizing resources is also greatly improved (He et al., 2020). The flow of products is tracked at every level of the supply chain, and all information related to parts and products is entered into the system and uploaded for managers (Abdel-Basset et al., 2018). The IoT optimizes the scheduling of the production process and provides deep integration of the physical production process and information system, which accelerates transformation and updating, reduces production costs, decreases energy consumption, and promotes the manufacturing industry for globalization and credibility (Huang, 2020).

By employing the IoT, defective products are identified in the production, storage, and transportation processes. In addition, RFID allows companies to track products and easily perform product recalls, thereby reducing recall costs as well (Yan , 2017). With the IoT, all transportation information will be available to the entire supply chain using smart objects. This increases the likelihood of monitoring and saving goods, minimizes return costs, and has a significant impact on customer satisfaction (Abdel-Basset et al., 2018). The emergence of the IoT, as a new revolutionary technology in the field of information technology, has provided the possibility of creating major transformations in supply chain management (Al-Fuqaha et al., 2015).

The IoT, by connecting physical objects to the network and enabling the collection and exchange of information in real time, leads to increased transparency, accuracy, and facilitation of affairs in the supply chain. Organizations can use this intelligent data to receive early warnings and identify internal and external positions to improve processes. The application of the IoT in the supply chain results in increased flexibility and responsiveness of the chain, reduced ordering time, reduced inventory levels, and reduced instances of shortages. This technology has good potential for improving the performance of the intelligent supply chain and integrates the flow of materials, information, and capital with integrated goals.

Several studies have examined the benefits of the IoT in the supply chain. For example, studies have shown that the IoT helps organizations remain in the competitive market and serve customers with appropriate inventory levels without the need for warehousing, increasing customer satisfaction. RFID technology, as one of the key IoT technologies, provides the possibility of identification, tracking, and information transfer at high speed, and it is an effective tool for solving the problem of inventory movement. Additionally, Wireless Sensor Networks (WSN), which consist of a large number of sensor nodes scattered in the environment, are used for monitoring and detecting objects and individuals, and they can collaborate with RFID tags. Cloud Computing, as an internet-based platform, also provides the possibility of processing a huge volume of data generated by IoT devices at high speed and is very efficient for real-time decision making.

Despite the advancements, the speed of processing decisions in conditions of uncertainty is still an area with high potential for improvement. The IoT, by producing a huge volume of data in real time, requires efficient information processing systems for automatic decision making. The inability of the

IoT to properly address security issues can limit its development. Blockchain technology has been proposed as a solution to increase security, transparency, and reliability in the supply chain, especially in combination with the IoT. The use of blockchain can take distribution transparency to a new level and solve the security issues of the IoT (Pathak et al., 2007).

5-4 Uncertainty

In real-world conditions of many industrial and production environments, there are a number of uncertain parameters whose precise estimation is difficult. On the other hand, the efficiency of a mathematical model largely depends on the accuracy of estimating the input parameters used in that model; therefore, the presence of uncertainty in the model's parameters and neglecting this issue can pose a major challenge for making appropriate decisions regarding the model's variables. The solutions obtained from optimization problems are significantly sensitive to disturbances in the problem's parameters (Ben-Tal & Nemirovski, 2000).

In other words, changing the values of input parameters from the predicted value can affect the optimality and feasibility of problems and lead to sub-optimality or even infeasibility of the problem. Therefore, in recent years, extensive research has been conducted to consider data uncertainty in mathematical models. Researchers have used various approaches to deal with this uncertainty, which are briefly described below:

Fuzzy Programming: For modeling and solving optimization problems in which parameters, such as demand, costs, times, and capacities, are expressed uncertainly or ambiguously (Gitinavard et al., 2024; Mula et al., 2010; Nemati et al., 2017).

Triangular Fuzzy Numbers (TFNs): A common method for representing ambiguous operational costs due to computational simplicity (Nemati et al., 2017).

Robust Optimization (RO): An approach to handling uncertain parameters, such as production processing time and demand, to find near-optimal solutions that are acceptable even in the worst-case scenario (Rezaei & Liu, 2024; Suryawanshi & Dutta, 2022).

Stochastic Programming: For modeling and solving optimization problems that involve random elements and consider multiple probable scenarios (Mula et al., 2010).

In the following, we describe two common approaches: stochastic programming and robust optimization, which have been used in previous research to confront uncertainty in parameters.

Robust optimization and possibilistic programming as methods for modeling uncertainty in parameters such as production processing time and demand have received attention (Rezaei & Liu , 2024; Suryawanshi et al., 2022).

1-5-4 Scenario-Based Stochastic Programming

In this method, which is one of the traditional ways to address uncertainty, several different scenarios for the input parameters are considered, each of which may occur with different probabilities. This method has two fundamental flaws that affect its application:

1- Estimating the probability distribution for each of the input parameters of the problem is very difficult.

2- The size of the optimization model increases dramatically due to the large number of scenarios. Therefore, in this method, we will face severe computational challenges.

2-5-4 Robust Optimization Approach

Robust optimization is a modern approach to optimization under uncertainty, in which the mathematical model is non-probabilistic and the parameters are deterministic but represented as sets. In optimization problems, the best estimates of data, referred to as nominal data, are usually used in mathematical models. In this approach, instead of making solutions insensitive to random uncertainty across a number of probabilistic scenarios, the decision maker produces a solution that is optimal with respect to any uncertainty within a given set of data. In other words, the objective of robust optimization is to reduce the financial loss caused by deviations of input parameters from their nominal values. In this method, the worst possible case of the parameters is optimized using a minimax objective function.

The reasons for the growing attention to this approach are presented below. These reasons have led to the success and increasing application of robust optimization in many practical fields (Ben-Tal et al., 2009).

1. The intrinsic attractiveness of the approach due to providing an appropriate concept for handling parameter uncertainty in many real-world applications.
2. The simplicity of the approach in terms of computational size and complexity.

According to studies by Morabito, robust optimization is one of the approaches that performs very efficiently in situations involving uncertainty. Robust optimization was first introduced in 1973 by Soyster. The model proposed by Soyster is highly conservative and represents a pessimistic approach. Over the past two decades, extensive efforts have been made to develop tractable robust models that are suitable for solving various optimization problems with uncertain data. Ben-Tal and Nemirovski proposed models whose robust counterparts of linear programming problems are second-order cone programming models. These models are less conservative and yield better solutions. Meanwhile, Bertsimas and Sim (2004) brought a major transformation to robust optimization. In their proposed model, the degree of conservatism is adjustable, and the robust counterpart of the original problem remains a linear programming problem. This model can also be applied to optimization problems with discrete variables.

1-2-5-4 Interval Robust Optimization

In the approach proposed by Bertsimas and Sim (2004), it is assumed that in the following model, constraint i contains $[\Gamma_i]$ uncertain technological coefficients, and uncertainty exists in matrix A. The uncertain coefficients $\tilde{a}_{ij}$ take values in the interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$ and there is exactly one uncertain parameter such as a_{it} that deviates from its nominal value by $\Gamma_i - [\Gamma_i]$.

$Max\ Z = cx$.
$\tilde{a}x \le b$ (*)
$l \le x \le u$

If $\beta_i(x^*, \Gamma_i)$ is defined as the maximum amount of deviation caused by Γ_i coefficient deviations, the above model can be rewritten as:

$Max\ Z = cx$.
$\tilde{a}x + \beta_i(x^*, \Gamma_i) \le b$ (**)
$l \le x \le u$

Based on the definition of $\beta_i(x^*, \Gamma_i)$, it can be obtained as follows. Γ_i is a positive real number in the interval $[0, |J_i|]$, where J_i is the set of uncertain coefficients in constraint i:

$\beta_i(x^*, \Gamma_i) = Max\ \sum_{j \in J_i} \hat{a}_{ij} |x_j^*| z_{ij}$ (***)
s.t.
$$\sum_{j \in J_i} z_{ij} \le \Gamma_i \quad \forall i$$
$0 \le z_{ij} \le 1 \quad \forall j \in J_i$

Here, z_{ij} represents the degree of deviation of coefficient a_{ij} from its nominal value for uncertain coefficients. It is a number between zero and one, and the sum of these deviations over all uncertain coefficients is bounded above by the selected uncertainty budget Γ_i.

Since the optimal solution of model (***) is equal to that of its dual problem, to preserve the linearity of model (**), while ensuring optimality and feasibility of the solution in the presence of uncertain parameter deviations, Bertsimas and Sim (2004) developed the following robust counterpart by substituting the dual of $\beta_i(x^*, \Gamma_i)$ into the original problem:

$Max\ Z = cx \quad (****)$.

s.t.

$$\sum_{m} \bar{a}_{ij}x_j + z_i\Gamma_i + \sum_{j\in J_i} p_{ij} \ \le b_i \quad \forall i$$
$$z_i + p_{ij} \ \ge \hat{a}_{ij}y_j \quad \forall(i\ ,j) \in J_i \ \text{(ii)}$$
$$-y_j \le x \le y_j \quad \forall j \qquad \text{(iii)}$$
$$l_j \le x_j \le u_j \quad \forall j$$
$$z_i, p_{ij}\ , y_j \ge 0$$

In this model, z_i, p_{ij} are the corresponding dual variables. If x_j is a non-negative variable, constraints (ii) and (iii) can be merged by substituting x_j for y_j. The parameter Γ_i controls the degree of conservatism of the model. The above robust model is a linear programming problem since linear programming problems can be readily solved using standard optimization packages.

Furthermore, if in the original problem (*), some variables are restricted to be integers, the robust counterpart (****) preserves similar properties. That is, if the original problem is a mixed-integer programming problem, its robust counterpart is also a mixed-integer programming problem. By extending the concept of the uncertainty budget, Bertsimas and Sim (2004) provided the decision maker with a flexible trade-off between model performance and robustness. The uncertainty budget represents the total deviation of uncertain parameters from their nominal values and reflects the model's ability to maintain feasibility and proximity to optimality under different levels of uncertainty (Bertsimas et al., 2011). In the present study, their proposed approach is used to account for uncertainty in demand, market return rate, and transportation costs.

3-5-4 Managerial Implications

This research can be beneficial both theoretically and practically for students and researchers in the field of virtual supply chains. It can also be applied to optimization problems in the presence of uncertain parameters. Moreover, the use of the IoT in supply chains enhances the responsiveness and efficiency of the supply chain in dealing with various managerial challenges. When data generated through IoT are efficiently collected and analyzed, they can provide valuable information about different aspects of the supply chain and issue warnings regarding current conditions that require adjustment or improvement. A timely and appropriate response to these warnings can significantly improve supply chain performance. IoT enables a reduction in the time between data collection and decision making, helping supply chains respond promptly to emerging changes, thereby increasing supply chain agility and responsiveness. Consequently, ensuring data security and the reliability of information exchanged among supply chain members is of critical importance.

Table 1) Summary of Researches

Author (Year)	Uncertain Quantity of Returned Products	Uncertain Transportation Cost (Others)	Uncertain Demand	Deterministic Quantity of Returned Products	Deterministic Transportation Cost	Deterministic Demand	Information Security	Technology Facilities	Multi-Period	Multi-Product	Internet of Things	Closed-Loop Supply Chain	Virtual Supply Chain	Simple Supply Chain
Pishvaee et al. (2010)												*		*

Pishvaee & Torabi (2010)	fuzzy*	fuzzy* Transportation	fuzzy*						*			*		*
Ramezani et al. (2014)	fuzzy*	fuzzy* Transportation	fuzzy*						*	*		*		*
Saffar & Razmi (2015)	fuzzy*	fuzzy* Transportation							*	*		*		*
Verdouw et al. (2013)					*	*							*	*
Zeballos et al. (2014)			probab* ilistic						*	*		*		*
Long (2014)					*	*							*	*
Fang et al. (2015)				*	*	*			*		*	*		*
Yan (2017)					*	*		*		*	*			*
Kuli_nska & Kuli_nska (2019)					*	*							*	*
Yadav & Misra (2019)					*	*								*
Nishi et al. (2020)					*	*		*					*	*
Matsuda et al. (2020)					*	*							*	*
Sallam et al. (2023)				*	*	*	*	*			*	*	*	*
Vlachos & Graham (2025)					*	*	*	*			*	*	*	*
Current Research	inter* val	interval*	interva* l				*	*	*	*	*	*	*	

Mathematical Model Notations

Sets:

- s: Set of suppliers, s = 1,2,…,S
- r: Set of product components, r = 1,2,…,R
- p: Set of product types, p = 1,2,…,P
- m: Set of production centers, m = 1,2,…,M
- d: Set of distribution centers, d = 1,2,…,D
- n: Set of customers, n = 1,2,…,N
- x: Set of collection centers, x = 1,2,…,X
- t: Set of time periods, t = 1,2,…,T
- b: Set of recycling centers, b = 1,2,…,B
- z: Set of disposal centers, z = 1,2,…,Z

- i: Auxiliary index (I = 1,2,3,4,8,17)
- j: Auxiliary index (j = 1)
- k: Auxiliary index (k = 8)

Parameters (Ordinary Supply Chain):

- $\widetilde{dem}_{pnt}$: Customer demand for product p in period t
- $\tilde{C}_{xzt}$: Transportation cost from collection center x to disposal center z in period t
- $\tilde{C}_{bzt}$: Transportation cost from recycling center b to disposal center z in period t
- $\tilde{C}_{mzt}$: Transportation cost from production center m to disposal center z in period t
- $\tilde{C}_{bmrt}$: Transportation cost of component r from recycling center b to production center m in period t
- $\tilde{C}_{xbt}$: Transportation cost from collection center x to recycling center b in period t
- $\tilde{C}_{dnt}$: Transportation cost from distribution center d to customer n in period t
- $\tilde{C}_{dmt}$: Transportation cost from distribution center d to production center m in period t
- $\tilde{C}_{mdt}$: Transportation cost from production center m to distribution center d in period t
- μ**p**t : Product failure rate in the production process in period t
- TCC**p**t : Collection and classification cost of product p at collection center x in period t
- DX**p**t : Rate of recyclable products in period t
- MCit : Maximum capacity of center i in period t
- TBC**p**bt : Recycling cost of product p at recycling center b in period t
- L: Useful life of products
- θ : Average reward value offered to customers
- FCst : Fixed cost of ordering components from supplier s in period t
- re_{pt} : Maximum return rate from the market in period t
- $\widetilde{ror}_t$: Amount of returned products from customers in period t
- SSrmt : Safety stock of component r at manufacturing center m in period t
- HCrmt : Holding cost of component r at manufacturing center m in period t
- HC**p**xt : Holding cost of returned product p at collection center x in period t
- Prpt: Selling price of product p in period t
- PUCst : Purchase cost of components from supplier s in period t
- CPpmt : Processing cost of product p at manufacturing center m in period t
- DBZpt : Disposal ratio at recycling center b in period t
- FCm : Fixed cost of opening production center m
- FCd : Fixed cost of opening distribution center d
- FCx : Fixed cost of opening collection center x
- FCb : Fixed cost of opening recycling center b
- FCz : Fixed cost of opening disposal center z
- TZCrt : Disposal cost of component r at disposal center z in period t
- Scpt : Shortage cost of product p in period t
- Drpt : Reproduction ratio of product p in period t
- qrpt : Quantity of component r required for product p

- HCpmt : Holding cost of product p at manufacturing center m in period t
- refpt : Reproduction cost of product p in period t

Parameters (Uncertainty):

- $\overline{dem}_{pnt}$: Deterministic part of customer demand for product p in period t
- $\widehat{dem}_{pnt}$: Uncertain part of customer demand for product p in period t
- $\overline{ror}_t$: Deterministic part of the amount of returned products from customers in period t
- $\widehat{ror}_t$: Uncertain part of the amount of returned products from customers in period t
- $\bar{C}_{xzt}$: Deterministic part of transportation cost from collection center x to disposal center z in period t
- $\hat{C}_{xzt}$: Uncertain part of transportation cost from collection center x to disposal center z in period t
- $\bar{C}_{bzt}$: Deterministic part of transportation cost from recycling center b to disposal center z in period t
- $\hat{C}_{bzt}$: Uncertain part of transportation cost from recycling center b to disposal center z in period t
- $\bar{C}_{mzt}$: Deterministic part of transportation cost from production center m to disposal center z in period t
- $\hat{C}_{mzt}$: Uncertain part of transportation cost from production center m to disposal center z in period t
- $\bar{C}_{bmrt}$: Deterministic part of transportation cost of component r from recycling center b to production center m in period t
- $\hat{C}_{bmrt}$: Uncertain part of transportation cost of component r from recycling center b to production center m in period t
- $\bar{C}_{xbt}$: Deterministic part of transportation cost from collection center x to recycling center b in period t
- $\hat{C}_{xbt}$: Uncertain part of transportation cost from collection center x to recycling center b in period t
- $\bar{C}_{dnt}$: Deterministic part of transportation cost from distribution center d to customer n in period t
- $\hat{C}_{dnt}$: Uncertain part of transportation cost from distribution center d to customer n in period t
- $\bar{C}_{dmt}$: Deterministic part of transportation cost from distribution center d to production center m in period t
- $\hat{C}_{dmt}$: Uncertain part of transportation cost from distribution center d to production center m in period t
- $\bar{C}_{mdt}$: Deterministic part of transportation cost from production center m to distribution center d in period t
- $\hat{C}_{mdt}$: Uncertain part of transportation cost from production center m to distribution center d in period t
- Γ: Integer uncertainty budget parameter, ranging from zero to the number of uncertain parameters in each constraint

Decision Variables (Related to Uncertainty):

- p_{pnt}^{u}: Dual variables associated with uncertainty in $\widehat{dem}_{pnt}$
- p_{t}^{u}: Dual variables associated with uncertainty in $\widehat{ror}_{t}$
- p_{mdt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{mdt}$
- p_{dnt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{dnt}$
- p_{dmt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{dmt}$
- p_{xbt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{xBt}$
- p_{mzt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{mZt}$
- p_{bzt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{BZt}$
- p_{xzt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{XZt}$
- p_{bmrt}^{u}: Dual variables associated with uncertainty in $\hat{C}_{bmrt}$
- Z_1: Dual variable associated with uncertainty in the objective function

Decision Variables (Ordinary Supply Chain):

- Qrst: Quantity of component r purchased from supplier s in period t
- Qpmt: Quantity of product p produced at manufacturing center m in period t
- Qpzt: Quantity of returned product p disposed of at disposal center z in period t
- Xst: $\begin{cases} 1 & \text{if components are purchased from supplier } s \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
- rpt: Return rate of products returned by customers in period t
- Prbpt: Purchase price of returned products in period t
- IQpmt: Ending inventory of product p at manufacturing center m in period t
- Qrbt: Quantity of component r recovered from recycled products p at recycling center b in period t
- Qpmdt: Quantity of product p shipped from manufacturing center m to distribution center d in period t
- Qpdnt: Quantity of product p shipped from distribution center d to customer n in period t
- Qpxbt: Quantity of product p shipped from collection center x to recycling center b in period t
- Qpxzt: Quantity of product p shipped from collection center x to disposal center z in period t
- Ypnxt: Quantity of returned product p shipped from customer n to collection center x in period t
- X_{it}: $\begin{cases} 1 & \text{if facility i is opened in period t} \\ 0 & \text{otherwise} \end{cases}$
- λ: Scaling coefficient between the price and the return rate of products
- IQrmt: Ending inventory of component r at manufacturing center m in period t
- IQpxt: Ending inventory of returned product p at collection center x in period t
- ICt: Inventory holding costs of components and products at manufacturing centers and returned products at collection centers in period t
- FCt: Fixed costs of the closed-loop supply chain network in period t
- SHCt: Shortage cost in period t
- FSCt: Cost of the forward supply chain in period t
- RSCt: Cost of the reverse supply chain in period t

Parameters (Virtual Supply Chain):

- EMt: Maximum allowable energy consumption in period t
- Del_t^{tr}: Average network delay for data processing in period t
- Delt: Maximum allowable network delay in period t
- CSht: Damage cost caused by security threats in period t
- C_{mpt}^{Re}: Product recall cost of product p in the manufacturing section
- C_t^{tag}: Cost of purchasing RFID tags in period t
- CB_t^{tag}: Cost of recycling RFID tags in period t
- e_t^{tr}: Energy cost per unit for recording, processing, and transmitting data via IoT in period t
- P_m^{IoT}: Fixed IoT energy consumption cost at manufacturing center m
- P_d^{IoT}: Fixed IoT energy consumption cost at distribution center d
- P_x^{IoT}: Fixed IoT energy consumption cost at collection center x
- P_b^{IoT}: Fixed IoT energy consumption cost at recycling center b
- FC_m^{IoT}: Fixed cost of IoT facilities at manufacturing center m
- FC_d^{IoT}: Fixed cost of IoT facilities at distribution center d
- FC_x^{IoT}: Fixed cost of IoT facilities at collection center x
- FC_b^{IoT}: Fixed cost of IoT facilities at recycling center b
- B: Maximum budget allocated to virtual supply chain security
- δ: Recycling rate of RFID tags

Decision Variables (Virtual Supply Chain):

- $X_{mt}^{IoT}: \begin{cases} 1 & \text{if IoT facilities are employed at manufacturing center m in period t} \\ 0 & \text{otherwise} \end{cases}$
- $X_{dt}^{IoT}: \begin{cases} 1 & \text{if IoT facilities are employed at distribution center d in period t} \\ 0 & \text{otherwise} \end{cases}$
- $X_{xt}^{IoT}: \begin{cases} 1 & \text{if IoT facilities are employed at collection center x in period t} \\ 0 & \text{otherwise} \end{cases}$
- $X_{bt}^{IoT}: \begin{cases} 1 & \text{if IoT facilities are employed at recycling center b in period t} \\ 0 & \text{otherwise} \end{cases}$
- Pt: Probability of security threat occurrence in period t
- Vt: Investment in security in period t
- FCIt: Fixed cost of IoT facilities in period t
- Rect: Recall cost of the manufacturing section in period t
- CSt: IoT information security cost in period t
- TCt: RFID tags cost
- Et: Energy consumption in period t
- P_t^{IoT}: Fixed IoT energy consumption cost in period t
- f_t^{tr}: Data recorded, processed, and transmitted by IoT in period t

Mathematical Model of the Problem
Objective Functions:

We have two objective functions:

- First objective function: Maximizes the profit of the virtual supply chain.

$$Profit = Max\ Z = \sum_{p}\sum_{d}\sum_{n}\left(Q_{pdnt}\ pr_{pt}\right) - \left(FC_t + FSC_t + SC_t + RSC_t + IC_t + Rec_t + FCI_t + TC_t + E_t + CS_t\right) \quad \forall t$$

- Second objective function: Minimizes the amount of delay in data processing.

$$Delay\ in\ process\ datas = Min\ Z' = \sum_{t}\left(Del_t^{tr} \times f_t^{tr}\right)$$

Model Constraints:

$$FC_t = \sum_{s}(X_{st} \times FC_{st}) + \sum_{m}(X_{mt} \times FC_m) + \sum_{d}(X_{dt} \times FC_d) + \sum_{x}(X_{xt} \times FC_x) + \sum_{b}(X_{bt} \times FC_b) + \sum_{z}(X_{zt} \times FC_z) \quad \forall t \quad (1)$$

$$FSC_t = \sum_{r}\sum_{s}(Q_{rst} \times PUC_{st}) + \sum_{p}\sum_{m}\left(Q_{pmt} \times CP_{pmt}\right) + \sum_{p}\sum_{m}\sum_{d}\left(Q_{pmdt} \times \tilde{C}_{mdt}\right) + \sum_{p}\sum_{d}\sum_{n}\left(Q_{pdnt} \times \tilde{C}_{dnt}\right) \quad \forall t \quad (2)$$

$$SC_t = \left(\widetilde{dem}_{pnt} - \sum_{d} Q_{pdnt}\right) \times Sc_{pt} \quad \forall p, n, t \quad (3)$$

$$RSC_t = \sum_{p}\sum_{m}\sum_{d}\left(Q_{pmdt} \times Dr_{pt}(ref_{pt} + \tilde{C}_{dmt})\right) + \sum_{b}\sum_{p}\sum_{n}\sum_{x}\left[Y_{pnxt}\left(TCC_{pt} + Pr_{bpt}\right)\right] + \sum_{p}\sum_{x}\sum_{b}\left(Q_{pxbt}\left(\tilde{C}_{xbt} + TBC_{pbt}\right)\right) + \sum_{r}\sum_{b}\sum_{m}(Q_{rbt} \times C_{bmrt}) + \sum_{p}\sum_{m}\sum_{r}\sum_{z}\left[\left(Q_{pmt} \times \mu_{pt} \times q_{rpt}(\tilde{C}_{mzt} + TZC_{rt})\right)\right] + \sum_{p}\sum_{x}\sum_{b}\sum_{r}\sum_{z}\left[\left(Q_{pxbt} \times q_{rpt} \times DBZ_{pt}\right)(\tilde{C}_{bzt} + TZC_{rt})\right] + \sum_{p}\sum_{x}\sum_{z}\sum_{r}\left[\left(Q_{pxzt} \times q_{rpt}(\tilde{C}_{xzt} + TZC_{rt})\right)\right] \quad \forall t \quad (4)$$

$$IC_t = \sum_{r}\sum_{m}(HC_{rmt} \times IQ_{rmt}) + \sum_{p}\sum_{m}\left(HC_{pmt} \times IQ_{pmt}\right) + \sum_{p}\sum_{x}\left(HC_{pxt} \times IQ_{pxt}\right) \quad \forall t \quad (5)$$

$$r_{pt} = (1 - \lambda) \times re_{pt} \quad \forall p, t \quad (6)$$

$$\lambda = e^{-\frac{pr_{bpt}}{\theta}} \quad \forall b, p, t \quad (7)$$

$$\sum_{n}\sum_{x} Y_{pnxt} = \widetilde{ror}_t \times \sum_{n}\sum_{d} Q_{pdn(t-L)} \quad , \quad \forall p, \forall t > L \quad (8)$$

$$IQ_{rmt} = SS_{rmt} + IQ_{rm(t-1)} + \sum_{s} Q_{rst} + \sum_{b} Q_{rbt} - \sum_{p}(Q_{pmt} \times q_{rpt}) \quad \forall m, r, t \quad (9)$$

$$IQ_{pmt} = IQ_{pm(t-1)} + Q_{pmt} + \sum_{d} (Q_{pmd(t-1)} \times Dr_{pt}) - \sum_{d} Q_{pmdt} \quad \forall p, m, t \quad (10)$$

$$IQ_{pxt} = IQ_{px(t-1)} + \sum_{n} Y_{pnxt} - \sum_{b} Q_{pxbt} - \sum_{z} Q_{pxzt} \quad \forall p, x, \forall t > L \quad (11)$$

$$\sum_{b} Q_{pxbt} = \sum_{n} Y_{pnxt} \times DX_{pt} \quad \forall p, x, t \quad (12)$$

$$\sum_{z} Q_{pxzt} = \sum_{n} Y_{pnxt} \, (1 - DX_{pt}) \quad \forall p, x, t \quad (13)$$

$$Q_{rbt} = \sum_{p} \sum_{x} \left(q_{rpt} \times Q_{pxbt} \, (1 - DBZ_{pt}) \right) \quad \forall r, b, \forall t > L \quad (14)$$

$$Q_{rbt} = 0 \quad \forall r, b, \forall t \le L \quad (15)$$

$$\sum_{m} Q_{pmt} \times q_{rpt} = \sum_{s} Q_{rst} + \sum_{b} Q_{rbt} \quad \forall p, r, t \quad (16)$$

$$\sum_{m} Q_{pmt} \le \sum_{n} \widetilde{dem}_{pnt} \quad \forall p, t \quad (17)$$

$$\sum_{m} Q_{pmdt} \ge \sum_{n} Q_{pdnt} \quad \forall p, d, t \quad (18)$$

$$\sum_{d} Q_{pmdt} = Q_{pmt} \, (1 - \mu_{pt}) \quad \forall p, m, t \quad (19)$$

$$\sum_{n} Q_{pdnt} = \sum_{m} Q_{pmdt} \left(1 - Dr_{pt}\right) \quad \forall p, d, t \quad (20)$$

$$\sum_{d} Q_{pdn(t-L)} \ge \sum_{x} Y_{pnxt} \quad \forall p, n, \forall t > L \quad (21)$$

$$Q_{rst} \le MC_{rst} \quad \forall r, s, t \quad (22)$$

$$Q_{pmt} \le MC_{pmt} \quad \forall p, m, t \quad (23)$$

$$\sum_{m} Q_{pmdt} \le MC_{pdt} \quad \forall p, d, t \quad (24)$$

$$\sum_{n} Y_{pnxt} \le MC_{pxt} \quad \forall p, x, t \quad (25)$$

$$\sum_{x} Q_{pxbt} \le MC_{pbt} \quad \forall p, b, t \quad (26)$$

$$\sum_{n} \sum_{x} \left(Y_{pnxt}\right) (1 - DX_{pt}) + \sum_{x} \sum_{b} Q_{pxbt} \times DBZ_{pt} \le \sum_{z} MC_{pzt} \quad \forall p, t \quad (27)$$

$$X_{mt} \ge 1 \quad \forall m, \forall t \quad (28)$$

$$X_{dt} \ge 1 \quad \forall d, \forall t \quad (29)$$

$$X_{xt} \ge 1 \quad \forall x, \forall t \quad (30)$$

$$X_{bt} \ge 1 \quad \forall b, \forall t \quad (31)$$

$$X_{zt} \ge 1 \quad \forall z, \forall t \quad (32)$$

$$X_{m(t+1)} - X_{mt} \ge 0 \quad \forall m, \forall t \quad (33)$$

$$X_{d(t+1)} - X_{dt} \ge 0 \quad \forall d, \forall t \quad (34)$$

$$X_{x(t+1)} - X_{xt} \geq 0 \quad \forall x, \forall t \quad (35)$$

$$X_{b(t+1)} - X_{bt} \geq 0 \quad \forall b, \forall t \quad (36)$$

$$X_{z(t+1)} - X_{zt} \geq 0 \quad \forall z, \forall t \quad (37)$$

$$Q_{rst}, Q_{pmt}, Q_{pmdt}, Q_{pdnt}, Y_{pnxt}, Q_{pxbt}, Q_{rbt} \geq 0 \quad \forall r, s, p, d, m, x, b, n, t \quad (38)$$

$$X_{mt}, X_{dt}, X_{xt}, X_{bt}, X_{zt} \in \{0,1\} \quad \forall d, m, x, b, z, t$$

$$X_{st} = \begin{cases} 0 & Q_{rst} = 0 \\ 1 & Q_{rst} > 0 \end{cases} \quad (39)$$

$$Rec_t = \sum_m \sum_p (\mu_{pt}\, C_{mpt}^{Re}\, Q_{pmt}) \quad \forall t \quad (40)$$

$$FCI_t = \sum_m (X_{mt}^{IoT} \times FC_m^{IoT}) + \sum_d (X_{dt}^{IoT} \times FC_d^{IoT}) + \sum_x (X_{xt}^{IoT} \times FC_x^{IoT}) + \sum_b (X_{bt}^{IoT} \times FC_b^{IoT}) \quad \forall t \quad (41)$$

$$X_{mt}^{IoT} \leq X_{mt} \quad \forall t, \forall m \quad (42)$$

$$X_{dt}^{IoT} \leq X_{dt} \quad \forall t, \forall d \quad (43)$$

$$X_{xt}^{IoT} \leq X_{xt} \quad \forall t, \forall x \quad (44)$$

$$X_{bt}^{IoT} \leq X_{bt} \quad \forall t, \forall b \quad (45)$$

$$X_{mt}^{IoT}, X_{dt}^{IoT}, X_{xt}^{IoT}, X_{bt}^{IoT} \in \{0,1\} \quad \forall d, m, x, b (46)$$

$$TC_t = \left[C_t^{tag} \Big(\sum_m \sum_d Q_{pmdt} - \sum_n \sum_x Y_{pnx(t-L)} \times \delta\Big) + \left(\sum_n \sum_x Y_{pnxt} \times \delta \times CB_t^{tag} \right) \right] \quad \forall p, t \quad (47)$$

$$P_t^{Iot} = \sum_m (X_{mt}^{IoT} \times P_m^{Iot}) + \sum_d (X_{dt}^{IoT} \times P_d^{Iot}) + \sum_x (X_{xt}^{IoT} \times P_x^{Iot}) + \sum_b (X_{bt}^{IoT} \times P_b^{Iot}) \quad \forall t \quad (48)$$

$$\begin{aligned} f_t^{tr} = {} & \sum_r \sum_s Q_{rst} + \sum_r \sum_b Q_{rbt} + \sum_p \sum_m Q_{pmt} + \sum_p \sum_m (Q_{pmt} \times \mu_{pt}) + \sum_p \sum_m \sum_d Q_{pmdt} \\ & + \sum_p \sum_m \sum_d (Q_{pmdt} \times Dr_{pt}) + \sum_p \sum_d \sum_n (Q_{pdnt}) + \sum_p \sum_n \sum_x (Y_{pnxt}) \\ & + \sum_p \sum_x \sum_b (Q_{pxbt}) + \sum_p \sum_x \sum_b (Q_{pxbt} \times DBZ_{pt}) \\ & + \sum_p \sum_x \sum_z (Q_{pxzt}) \quad \forall t \quad (49) \end{aligned}$$

$$Energy\ consumption = E_t = (e_t^{tr} \times f_t^{tr}) + P_t^{Iot} \quad \forall t \quad (50)$$

$$E_t \leq EM_t \quad \forall t \quad (51)$$

$$CS_t = (CSh_t \times P_t + V_t) \quad \forall t \quad (52)$$

$$CS_t \leq B \quad \forall t \quad (53)$$

$$(Del_t^{tr} \times f_t^{tr}) \leq Del_t \quad \forall t \quad (54)$$

Robust Optimization Model of the Problem

Bertsimas et al. (2011), by developing the concept of uncertainty budget, provided the decision maker with a flexible choice from a spectrum of model performance and robustness. The uncertainty budget is essentially the sum of deviations of uncertain parameters from their nominal values and represents the model's performance in maintaining feasibility and closeness to optimality in the event of various levels of uncertainty. This approach is also used to account for uncertainty in demand, market return rate, and transportation costs. By converting the objective function into a constraint and replacing the following constraints in place of constraints (2), (3), (4), (8), and (17), the robust model of the problem is obtained.

$Max\ f$

s.t.

$$
\begin{aligned}
f \leq \sum_{p}\sum_{d}\sum_{n}(Q_{pdnt}\ pr_{pt}) & \\
& -\Bigg(FC_t \\
& + \sum_{r}\sum_{s}(Q_{rst} \times PUC_{st}) + \sum_{p}\sum_{m}(Q_{pmt} \times CP_{pmt}) \\
& + \sum_{p}\sum_{m}\sum_{d}(Q_{pmdt} \times \bar{C}_{mdt}) + \sum_{p}\sum_{d}\sum_{n}(Q_{pdnt}\,\bar{C}_{dnt}) \\
& + \left(\overline{dem}_{pnt} - \sum_{d} Q_{pdnt}\right) \times Sc_{pt} \\
& + \sum_{p}\sum_{m}\sum_{d}\left(Q_{pmdt} \times Dr_{pt} \times \left(ref_{pt} + \bar{C}_{dmt}\right)\right) \\
& + \sum_{p}\sum_{n}\sum_{x} Y_{pnxt}\left(TCC_{pt} + Pr_{bpt}\right) \\
& + \sum_{p}\sum_{x}\sum_{b}\left(Q_{pxbt}\left(\bar{C}_{xbt} + TBC_{pbt}\right)\right) + \sum_{r}\sum_{B}\sum_{m}(Q_{rbt} \times \bar{C}_{bmrt}) \\
& + \sum_{p}\sum_{m}\sum_{r}\sum_{z}\left[\left(Q_{pmt} \times \mu_{pt} \times q_{rpt}(\bar{C}_{mzt} + TZC_{rt})\right)\right] \\
& + \sum_{p}\sum_{x}\sum_{b}\sum_{r}\sum_{z}\left[(Q_{pxbt} \times q_{rpt} \times DBZ_{pt})(\bar{C}_{bzt} + TZC_{rt})\right] \\
& + \sum_{p}\sum_{x}\sum_{z}\sum_{r}\left[\left(Q_{pxzt} \times q_{rpt}(\bar{C}_{xzt} + TZC_{rt})\right)\right] + IC_t + Rec_t + FCI_t + TC_t + E_t \\
& + CS_t\Bigg) + \Gamma_1 z_1 + \sum_{p}\sum_{n}\sum_{t}(p^u_{pnt}) + \sum_{m}\sum_{d}\sum_{t}(p^u_{mdt}) + \sum_{d}\sum_{n}\sum_{t}(p^u_{dnt}) \\
& + \sum_{d}\sum_{m}\sum_{t}(p^u_{dmt}) + \sum_{x}\sum_{b}\sum_{t}(p^u_{xbt}) + \sum_{m}\sum_{z}\sum_{t}(p^u_{mzt}) + \sum_{b}\sum_{z}\sum_{t}(p^u_{bzt}) \\
& + \sum_{x}\sum_{z}\sum_{t}(p^u_{xzt}) + \sum_{b}\sum_{m}\sum_{r}\sum_{t}(p^u_{bmrt})
\end{aligned}
$$

$$p^u_{pnt} + Z_1 \geq \hat{d}em_{pnt}\,Sc_{pt} \quad \forall p, n, t$$

$$p^u_{mdt} + Z_1 \geq Q_{pmdt} \times \hat{C}_{mdt} \quad \forall p, m, d, t$$

$$p^u_{dnt} + Z_1 \geq Q_{pdnt}\,\hat{C}_{dnt} \quad \forall p, d, n, t$$

$$p^u_{dmt} + Z_1 \geq \left(Q_{pmdt} \times Dr_{pt} \times \hat{C}_{dmt}\right) \quad \forall p, m, d, t$$

$$p^u_{xbt} + Z_1 \geq \left(Q_{pxbt}\,\hat{C}_{xbt}\right) \quad \forall p, x, b, t$$

$$p^u_{bmrt} + Z_1 \geq \left(Q_{rbt}\,\hat{C}_{bmrt}\right) \quad \forall b, m, r, t$$
$$p^u_{mzt} + Z_1 \geq \left(Q_{pmt} \times \mu_{pt} \times q_{rpt}\hat{C}_{mzt}\right) \quad \forall r, p, m, z, t$$
$$p^u_{bzt} + Z_1 \geq \left[\left(Q_{pxbt} \times q_{rpt} \times DBZ_{pt}\right)\left(\hat{C}_{bzt}\right)\right] \quad \forall r, p, x, b, z, t$$
$$p^u_{xzt} + Z_1 \geq \left(Q_{pxzt} \times q_{rpt} \times \hat{C}_{xzt}\right) \quad \forall r, p, x, z, t$$

$$\sum_r \sum_s (Q_{rst} \times PUC_{st}) + \sum_p \sum_m \left(Q_{pmt} \times CP_{pmt}\right) + \sum_p \sum_m \sum_d \left(Q_{pmdt} \times \tilde{C}_{mdt}\right) + \sum_p \sum_d \sum_n \left(Q_{pdnt}\,\tilde{C}_{dnt}\right) = FSC_t \quad \forall t \quad (2)$$

$$\sum_s \sum_r (Q_{rst} \times PUC_{st}) + \sum_p \sum_m \left(Q_{pmt} \times CP_{pmt}\right) + \sum_p \sum_m \sum_d \left(Q_{pmdt} \times \bar{C}_{mdt}\right) + \Gamma_2 Z_2 + p^u_{mdt} + \sum_p \sum_d \sum_n \left(Q_{pdnt}\,\bar{C}_{dnt}\right) + \Gamma_2 Z_2 + p^u_{dnt} = FSC_t \quad \forall t$$

$$\left(\widetilde{dem}_{pnt} - \sum_d Q_{pdnt}\right) \times Sc_{pt} = SC_t \quad \forall t, n, p \quad (3)$$

$$Sc_{pt}\,\overline{dem}_{pnt} + \Gamma_3 Z_3 + \sum_p \sum_t p^u_{pnt} - Sc_{pt} \sum_d Q_{pdnt} = SC_t$$

$$\begin{aligned} RSC_t = {} & \sum_p \sum_m \sum_d \left(Q_{pmdt} \times Dr_{pt}(ref_{pt} + \tilde{C}_{dmt})\right) + \sum_p \sum_n \sum_x Y_{pnxt}\left(TCC_{pt} + Pr_{bpt}\right) \\ & + \sum_p \sum_x \sum_b \left(Q_{pxbt}\,(\tilde{C}_{xbt} + TBC_{pbt})\right) + \sum_r \sum_b \sum_m \left(Q_{rbt} \times \tilde{C}_{bmrt}\right) \\ & + \sum_p \sum_m \sum_r \sum_z \left[\left(Q_{pmt} \times \mu_{pt} \times q_{rpt}(\tilde{C}_{mzt} + TZC_{rt})\right)\right] \\ & + \sum_p \sum_x \sum_b \sum_r \sum_z \left[\left(Q_{pxbt} \times q_{rpt} \times DBZ_{pt}\right)(\tilde{C}_{bzt} + TZC_{rt})\right] \\ & + \sum_p \sum_x \sum_z \sum_r \left[\left(Q_{pxzt} \times q_{rpt}(\tilde{C}_{xzt} + TZC_{rt})\right)\right] \quad \forall t \quad (4) \end{aligned}$$

$$\begin{aligned} RSC_t = {} & \sum_p \sum_m \sum_d \left(Q_{pmdt} \times Dr_{pt} \times (ref_{pt} + \bar{C}_{dmt})\right) + \Gamma_4 Z_4 + p^u_{dmt} \\ & + \sum_p \sum_n \sum_x Y_{pnxt}\left(TCC_{pt} + Pr_{bpt}\right) \\ & + \sum_p \sum_x \sum_b \left(Q_{pxbt}\,(\bar{C}_{xbt} + TBC_{pbt})\right) + \Gamma_4 Z_4 + p^u_{xbt} + \sum_r \sum_b \sum_m (Q_{rbt} \times \bar{C}_{bmrt}) \\ & + \Gamma_4 Z_4 + p^u_{bmrt} \\ & + \sum_p \sum_m \sum_r \sum_z \left[\left(Q_{pmt} \times \mu_{pt} \times q_{rpt}(\bar{C}_{mzt} + TZC_{rt})\right)\right] + \Gamma_4 Z_4 + p^u_{mzt} \\ & + \sum_p \sum_x \sum_b \sum_r \sum_z \left[\left(Q_{pxbt} \times q_{rpt} \times DBZ_{pt}\right)(\bar{C}_{bzt} + TZC_{rt})\right] + \Gamma_4 Z_4 + p^u_{bzt} \\ & + \sum_p \sum_x \sum_z \sum_r \left[\left(Q_{pxzt} \times q_{rpt}(\bar{C}_{xzt} + TZC_{rt})\right)\right] + \Gamma_4 Z_4 \\ & + p^u_{xzt} \quad \forall t \quad (4) \end{aligned}$$

$$\sum_{n}\sum_{x} Y_{pnxt} = \widetilde{ror}_t \times \sum_{n}\sum_{d} Q_{pdn(t-L)} \quad , \quad \forall p, \forall t > L \quad (8)$$

$$\sum_{n}\sum_{x} Y_{pnxt} = \overline{ror}_t \times \sum_{n}\sum_{d} Q_{pdn(t-L)} + \Gamma_8 W_8 + \sum_{t} p_t^u \quad \forall p, \forall t > L$$

$$W_8 + p_t^u \geq \widehat{ror}_t \times \sum_{n}\sum_{d} Q_{pdn(t-L)}$$

$$\sum_{m} Q_{pmt} \leq \sum_{n} \widetilde{dem}_{pnt} \quad \forall p, t \quad (17)$$

$$\sum_{m} Q_{pmt} \leq \sum_{n} \overline{dem}_{pnt} + \Gamma_{17} Z_{17} + \sum_{p}\sum_{t} p_{pnt}^u \quad \forall p, t$$

$$Z_1 + p_{pnt}^u \geq \widehat{dem}_{pnt}$$

$$p_{pnt}^u, p_{pt}^u, p_{mdt}^u, p_{mdt}^u, p_{dmt}^u, p_{xbt}^u, p_{mzt}^u, p_{bzt}^u, p_{xzt}^u, Z_1, Z_2, Z_3, Z_4, Z_{17}, W_8 \geq 0$$

Solution Method

After defining the conceptual model of the problem, a deterministic two-objective mathematical model was designed, and then, considering the uncertain parameters, the robust model was designed. For analyzing the information and checking the accuracy of the obtained answer, GAMS software has been used in small scales. To examine the correctness and proper functioning of the presented model and to prove its validity and application, a numerical example is provided.

5) Numerical Example and Findings

In this section, a numerical example is presented to demonstrate the performance of the deterministic and robust model presented in previous chapters and to prove its validity and application. Due to the lack of data in these models, the model was solved based on expert knowledge and data available in similar articles (Fang et al., 2015) by providing a numerical example. Considering that closed-loop supply chain optimization problems are in the category of NP-Hard problems (Mirghaderi & Modiri, 2021), and due to the complexity of the model, the computational time of exact solution methods is extremely high, and in most cases, it is not possible to solve such problems in real time. Factors causing the model's complexity include the large number of constraints and decision variables, as well as some variables being binary (zero and one). Furthermore, in multi-objective problems, the conflict between objectives adds to the problem's complexity. Therefore, the model presented in this research was solved in small dimensions using GAMS software. It is assumed that the producer produces five products, each consisting of different parts, for five time periods.

Data Generation Scheme

The parameter generation scheme presented in Table 2 includes all problem parameters, including both the ordinary supply chain and the virtual supply chain.

Table 2) The Parameter Generation Scheme

Generation Scheme	parameter	Generation Scheme	parameter	Generation Scheme	parameter
Uniform (1,20)	Del_t^{tr}	Uniform (0,4917)	SSrmt	Uniform (38,409)	$\boldsymbol{dem_{pnt}}$
Uniform (1,10000)	Delt	Uniform (0.05,0.05)	HCrmt	Uniform (1000,3500)	$\boldsymbol{C_{xzt}}$
Uniform (1300,3000)	CSht	Uniform (0.1,0.1)	HC**p**xt	Uniform (1000,3500)	$\boldsymbol{C_{bzt}}$
Uniform (5,6.5)	C_{mpt}^{Re}	Uniform (7000,10000)	Prpt	Uniform (1000,3500)	$\boldsymbol{C_{mzt}}$
Uniform (2,2)	C_t^{tag}	Uniform (8,18)	PUCst	Uniform (9,14)	$\boldsymbol{C_{bmrt}}$
Uniform (0.5,0.5)	CB_t^{tag}	Uniform (3,3)	CPpmt	Uniform (1000,3500)	$\boldsymbol{C_{xbt}}$
Uniform (0.1,0.1)	e_t^{tr}	Uniform (0.1,0.3)	DBZpt	Uniform (1000,3500)	$\boldsymbol{C_{dnt}}$

Uniform (50, 50)	P_m^{IoT}	Uniform (50000,80000)	FCm	Uniform (1000,3500)	$\boldsymbol{C_{dmt}}$
Uniform (50, 50)	P_d^{IoT}	Uniform (10000,15000)	FCd	Uniform (1000,3500)	$\boldsymbol{C_{mdt}}$
Uniform (50, 50)	P_x^{IoT}	Uniform (4000,8000)	FCx	Uniform (0.01,0.02)	µ**p**t
Uniform (50, 50)	P_b^{IoT}	Uniform (4000,8000)	FCb	Uniform (0.1,2)	TCC**p**t
Uniform (1000,5000)	FC_m^{IoT}	Uniform (4000,8000)	FCz	Uniform (0.90,0.95)	DX**p**t
Uniform (1000,5000)	FC_d^{IoT}	Uniform (0.2,4)	TZCrt	Uniform (10000,20000)	EMt
Uniform (1000,5000)	FC_x^{IoT}	Uniform (10,50)	Scpt	Uniform (12,20)	TBC**p**bt
Uniform (1000,5000)	FC_b^{IoT}	Uniform (0.1,0.4)	Drpt	Uniform (0.1,0.8)	Δ
Uniform (10000,14000)	MCpxt	Uniform (1000,10000)	B	Uniform (1,15)	Θ
Uniform (10000,14000)	MCpbt	Uniform (0.1,0.1)	HCpmt	Uniform (50,120)	FCst
Uniform (10000,19000)	MCrst	Uniform (2,2)	refpt	Uniform (0.6,0.9)	$\boldsymbol{re_{pt}}$
Uniform (30000,36000)	MCpmt			Uniform (0.6,0.8)	$\boldsymbol{ror_t}$
Uniform (25000,30000)	MCpdt			Uniform (1,4)	$\boldsymbol{q_{rpt}}$
Uniform (10000,14000)	MCpzt				

Computational Results

To examine the feasibility of the model, first, the deterministic model and the corresponding robust model solved in GAMS software. In order to study the impact of uncertainty in non-deterministic parameters on the objective functions, we have listed the parameters related to uncertainty in Table 3.

Table 3) Parameters Related to Uncertainty

۳=۷۵۰Γ	=۲۸/۸۶٪$\boldsymbol{\hat{C}_{dmt}}$	=۲۸/۸۶٪$\boldsymbol{\hat{C}_{mzt}}$	$\boldsymbol{\widehat{dem}_{pnt} = 28.86\%}$
۴=۱۹۵Γ	=۲۸/۸۶٪$\hat{C}_{mdt}$	=۲۸/۸٪$\hat{C}_{bmrt}$	=۲۸/۵٪$\boldsymbol{\widehat{ror}_t}$
۸=۵Γ	۱=۹۴۵Γ	=۲۸/۸۶٪$\hat{C}_{xbt}$	=۲۸/۸۶٪$\boldsymbol{\hat{C}_{xzt}}$
۱۷=۷۵۰Γ	۲=۱۵۵Γ	=۲۸/۸۶٪$\hat{C}_{dnt}$	=۲۸/۸۶٪$\boldsymbol{\hat{C}_{bzt}}$

In Table 4, the results of the objective functions after solving the aforementioned models are shown. Problems 1 and 2 correspond to the deterministic model and the robust model, respectively.

Table 4) Results of the Objective Functions

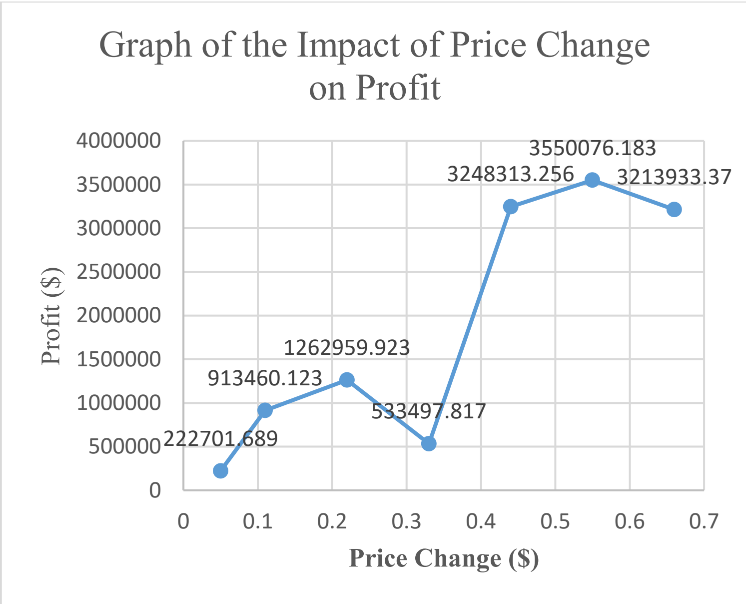
Processing Speed (Milliseconds)	Production Department Recall Cost	Tags Cost	Security Cost	Energy Cost	Total Profit ($)	Problem Number
۲۰۷۳۹/۷۸۹	۴۳۸/۴۸۳	۱۴۸۹/۲۶۱	۱۸۴۸۱/۳۳۶	۴۹۳/۲۵۵	۱۴۸۳۵۹/۸۵۵	۱
۱۶۲۱۱/۰۹۸	۲۱۹/۰۱۹	۱۰۹۵/۶۸۴	۳۶۳۲/۱۹۶	۹۶/۱۲۴	/۶۸۰ ۲۷۴۶۳۸۲	۲

The second problem, unlike the first one, is subject to uncertain parameters including customer demand, transportation costs, and return rates of returned products. Since the gamma coefficient in the objective function has the highest value, the profit has increased significantly. Here, the robust model, by considering uncertainty, generates more profit; however, due to the complexity of calculations and larger data volume, the processing delay time increases and creates more delay, while the deterministic model has less profit and shows lower processing delay.

Sensitivity Analysis

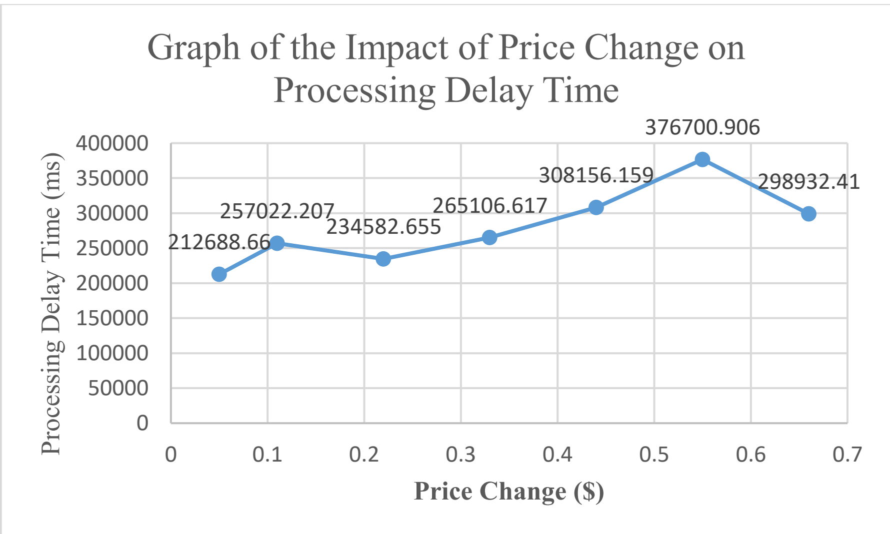
In this section, sensitivity analysis was conducted to examine the role of parameter changes on the objective functions and to verify the validity of the mathematical model. Initially, the role of changes in product prices on the objective functions, namely profit and the amount of delay created in data processing time, was examined. In Figure 1, the impact of increasing product prices from 5% to 66% on the supply chain profit is displayed.

Figure 1) Graph of the Impact of Price Change on Profit



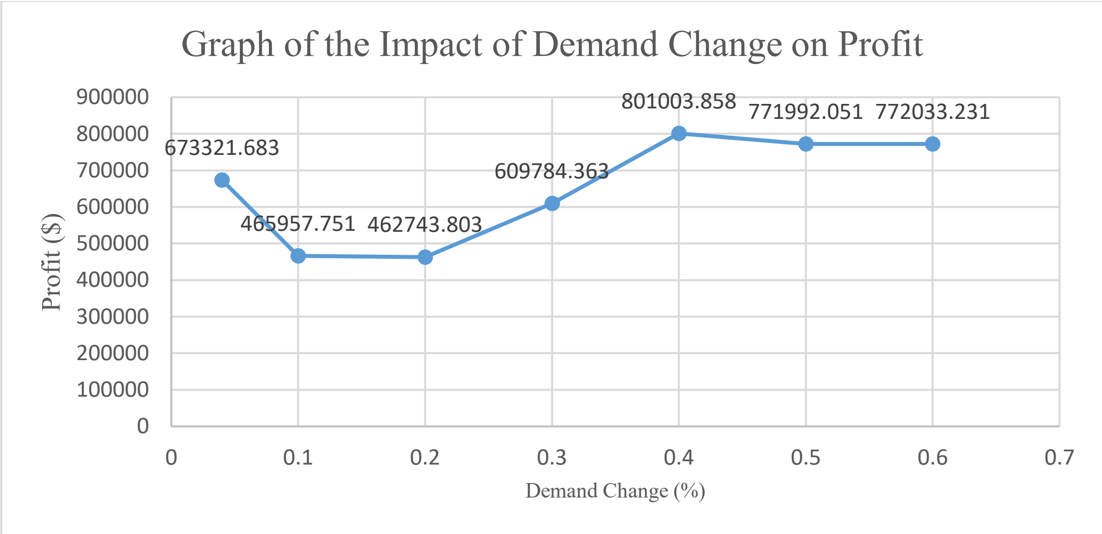
As can be seen in Figure 1, with the increase of the product prices, the revenue obtained from product sales increases; moreover, the supply chain profit increases significantly, with the graph showing an upward trend. In Figure 2, the impact of increasing product prices on the delay time in data processing is illustrated.

Figure 2) Graph of the Impact of Price Change on Processing Delay Time



In Figure 2, it is observed that with the increase in product prices from 5% to 66%, the amount of product production increases, and the delay time in data processing increases. Next, the role of changes in customer demand for products on the model's objective functions, namely profit and the amount of delay created in data processing time, has been examined. In Figure 3, the impact of increasing customer demand for products from 5% to 60% on the profit objective function is shown.

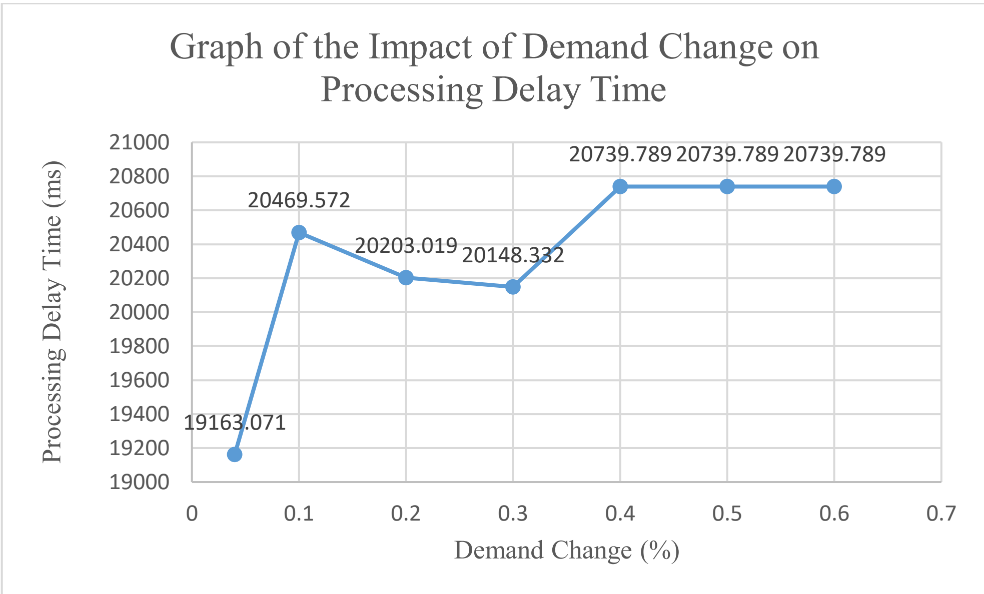
Figure 3) Graph of the Impact of Demand Change on Profit



According to Figure 3, overall, with the increase in demand, the supply chain profit increases. Because the increase in demand raises the amount of product production, more products are sold, and the supply chain revenue increases.

In Figure 4, the impact of increasing customer demand for products from 5% to 60% on the objective function of delay time in data processing was examined.

Figure 4) Graph of the Impact of Demand Change on Processing Delay Time



Based on Figure 4, overall, with the increase in demand, the delay time in data processing increases. As the demand raises, the amount of production increases, more products are labeled, the volume of data sent in the supply chain increases, and ultimately, the amount of delay in data processing increases.

6) Conclusion and Suggestions

In this study, a closed-loop virtual supply chain model under uncertainty is proposed using the IoT. The supply chain is a multi-period, multi-product network consisting of suppliers, manufacturers, distributors, customers, collection centers, recycling centers, and disposal centers. By considering two conflicting objective functions profit maximization and minimization of data processing delay and by employing a robust optimization approach, the proposed model demonstrates satisfactory performance under uncertainty in demand parameters, return rates, and transportation costs.

The costs considered in the model include facility opening costs for production plants and various centers, IoT infrastructure costs, ordering costs from suppliers, transportation costs, inventory holding costs, shortage costs, production operational costs, energy costs, and operational costs of IoT-enabled centers, among others. To manage uncertainty, the Bertsimas–Sim robust optimization approach is applied.

In the deterministic model, the two objectives profit and processing speed exhibit a direct relationship, mainly due to their mutual dependence on production volumes and the savings obtained from recycling materials and components.

The results indicate that the robust model, compared to the deterministic model, generates higher profit; however, due to the increased volume of data processing, it also experiences longer processing delay. Sensitivity analysis further confirms the direct impact of price and demand variations on both profit and processing time. The proposed model can therefore serve as an effective decision-support tool for supply chain managers operating under uncertainty.

In addition to operationalizing the virtual supply chain through IoT, this research also examines the modeling and optimization of both ordinary and virtual supply chains in a multi-period setting, aiming to support better decision-making under uncertain conditions. Given the rapid technological advancements in today’s dynamic environment and the need for quick responses to market and customer changes in highly competitive conditions, it is recommended that industry decision makers move their business activities toward virtualization to enhance overall business efficiency.

Based on the classification of reviewed and identified studies in the literature, the following directions are suggested for future research:

1. Incorporating shortage costs within a multi-criteria decision-making framework
2. Virtualization of the supply chain while addressing challenges such as IT infrastructure readiness, technical implementation issues, and organizational deployment challenges
3. Considering a probability distribution function for product deterioration rates
4. Extending the model by incorporating environmental and social sustainability criteria

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